How Markets Disrupt Mediated Trade

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Abstract

This paper studies markets with adverse selection and the degree to which intermediaries can foster efficient trade. I consider a setting in which a seller and buyer have interdependent values. Without any intermediation, the Lemons Problem guarantees that only the lowest type trades in any equilibrium. I consider an intermediary who brokers trade between the seller and the buyer by using a screening mechanism. When this is the *only* channel for trade, more efficient outcomes are possible in equilibrium, where higher types trade with positive probability. My main result, however, concludes that once the seller can *also* sell her asset without going through the intermediary, market failures re-emerge: trade of assets above the lowest quality shuts down in *both* the decentralized and mediated market. This paper shows that intermediation might be rendered completely ineffective when assets cannot be exclusively traded through the intermediary.

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1 Introduction

Adverse selection often causes markets to function inefficiently, and if severe, can result in market breakdowns, as has been known since Akerlof (1970). The key idea is that asymmetric information about the good being traded may prevent mutually beneficial trade, even if it is common knowledge that there are gains from trade.

These lost gains from trade can be recovered if an *intermediary* could broker trade between sellers (privately informed parties), and buyers (the uninformed party). Examples of such intermediaries include real estate agents, representing someone looking to sell a house, and stockbrokers in financial markets. Intermediaries can create a more efficient channel for sale by reducing the transaction costs associated with the sale, advertising, and even negotiating with potential buyers on behalf of the seller.

But how might such intermediaries solve the problem of adverse selection? The idea is that an intermediary can screen seller's types by offering her a a menu of contracts. The choices in the menu cause different types of the seller to *self-select* into different categories, and this *separation* helps mitigate adverse selection. To understand this, consider the real estate example. Suppose the real estate agent offers two possible choices to the seller. Either the seller could *only* consider selling at a high price, in which case, there is a chance that the property will remain unsold. Or, she could consider selling at a low price, it which case, it would almost definitely get sold.

Now, a seller with a high quality property might derive a a high benefit from living there herself, if a sale does not happen. So, she would be unwilling to sell at a low price. On the other hand, if the seller knows that there are are issues with her property (that might not be immediately observable to others), then she just wants to ensure a sale, even if its at a low price. So, in equilibrium, a high quality seller would choose the first option and a low quality seller would choose the second. The price at which the property is offered, therefore, acts as an endogenous signal of quality for a buyer who might otherwise not be able to observe all aspects of it perfectly. Since *only* the high quality properties are listed at higher prices, the buyer is willing to buy at these high prices. So, high quality sellers sell with positive probability in equilibrium, and the intermediary is able to mitigate the problem of adverse selection.[1]

[1] One might wonder why the property gets sold with a probability lower than one at the higher price, if the buyer is always willing to buy. There could be several interpretations for this. The interpretation I go with in this paper is that intermediary can *commit* to a lower probability of sale, to create a trade off for screening. However, an alternative interpretation could be that the intermediary has to "search" for a buyer; at a higher price, with some probability, the intermediary may not be able to *find* an appropriate buyer. No matter what the interpretation is, the idea is that when selling through the intermediary, at a

But these intermediaries do not operate in a vacuum. The seller often has other ways to sell, in case the intermediary is unable to broker trade. For example, if it looks like the sale is not going to happen through the agent, the property owner can list her property for sale herself, at a lower price. I model this *other* selling opportunity as a *static competitive market*, where trade takes place at a single, market clearing price.

Given that the seller has the option to sell through an intermediary or directly on the market herself, it's unclear the degree to which intermediation can address adverse selection. This question motivates my analysis: *how does the presence of a market disrupt intermediated trade?*

I consider a setting with *interdependent values* and severe adverse selection. There is a seller and a buyer. The seller has one unit of an indivisible good for sale. The quality of the good is the seller's *type*, and neither the intermediary, nor the buyer observes it. However, the distribution of types is common knowledge. The seller and the buyer have interdependent values for the good, and the buyer always values the good more than the seller. Even though the seller and buyer commonly know that there are gains from trade, the market suffers from a *lemons problem*: given the prior, the highest price that the buyer is willing to pay for the good is strictly lower that the reservation utility for the highest type of the seller.

The seller has two ways of selling the good: she can either sell through the intermediary, or on the market. The intermediary offers a menu of contracts, or *allocations*, where each allocation is a tuple (π, p). If the seller chooses this allocation, with probability π, she will get a chance to sell through the intermediary, at price p. The allocations could, therefore, have some uncertainty associated with them. For example, in (π, p), if $\pi < 1$, then, with probability $1 - \pi$, the seller will *not* get the chance to sell through the intermediary. On the market, sale happens at a single price p_M, which is determined in equilibrium.

The timeline is as follows: First, the intermediary offers a menu, and the seller chooses an allocation in that menu. Then, the uncertainty associated with this allocation is resolved: the seller either has the option of selling through the intermediary, or she doesn't. Finally, the seller decides whether to sell through the intermediary (if she has the option), or on the market. In particular, if the seller chooses an allocation and does *not* get the chance to get through the intermediary, she can, at this point, sell on the market.

The menu offered by the intermediary, therefore, induces a game where the seller chooses an allocation and then decides where to sell. I study the Perfect Bayesian Equilibria

higher price, there is some uncertainty about whether or not sale will take place, and this uncertainty is what enables screening.

of this game. Here, the intermediary has no direct control over the market. In equilibrium however, through the menu that it offers, it influences what types of the seller sell on the market, and therefore the market price. The market price, on the other hand, influences how the seller evaluates different choices in the menu, and therefore her choice of allocation.

I find that this equilibrium interaction between the market and the intermediary can completely destroy the intermediary's ability to screen, and therefore its ability to mitigate adverse selection. My main result, stated informally, is as follows:

> **Main Result**. *When the intermediary operates alongside the market, under some condition on parameters, the unique equilibrium outcome is a total breakdown of trade, where only the lowest type trades in equilibrium.*

Hence, the market may completely destroy the efficiency gains that come from intermediated trade. I show that this disruption occurs if and only there is a *lemons problem* for *every* subset of types at the bottom. So, if there are n possible types, consider any $k \leqslant n$. Conditional on the seller's type being in the set of the k *lowest types*, the buyer's expected value for the good is strictly lower than the reservation utility for the highest type in this set. I refer to this condition as the *Bottom Lemons Condition (BLC).*

The key idea is that when the intermediary is operating *alongside* the market, there is no way to deter the lowest type from "mimicking" the higher types' choice of allocation. And under the BLC, preventing this mimicking is essential for higher types to trade in equilibrium. To see this, suppose that the lowest and the second-lowest type, both sell at the same price in equilibrium. By the BLC, there is a *lemons problem* for the two lowest types. So conditional on this price, the buyer's expected value for the good is strictly lower than the reservation utility of the second-lowest type. So the buyer wouldn't buy.

When there is no market, and the intermediary is the *only* channel of sale, the lowest type can be deterred from mimicking the higher types' choice of allocation by making the probability of trade in these allocations lower. If the lowest type chooses these "high-price" allocations, with some probability, she might not be able to sell *at all.* But when the market is *also* present, the seller can sell on the market, in case sale does not happen through the intermediary. Therefore the lower probability of sale in the high-price allocations is no longer an effective deterrent, and the lowest type finds is optimal to mimic the second lowest type's choice of allocation.

This mimicking, as I argued, implies that neither the lowest, nor the second-lowest type can trade in equilibrium. BLC ensures that this unravelling continues, because *any* subset of types at the bottom suffers from the *lemons problem.* If the third lowest type trades with positive probability, *both* the lowest and second-lowest types would mimic the third lowest type's choice of allocation. But then, buyer's expected value conditional on this allocation's price is strictly lower than the reservation utility of the third lowest price.

Section 2 illustrates the main forces at work through a two-type example. Section 3 describes the general model, and Section 4 contains the main results, including the general result about how the market disrupts trade. Section 5 concludes.

1.1 Literature Review

The literature on market breakdowns due to adverse selection was initiated by Akerlof (1970). Akerlof considers a static, competitive market, where trade happens at a single, market clearing price. There is a large literature that takes a mechanism design approach to mitigating the breakdown problem, where there is an intermediary who screens different types of the seller by offering a menu of contracts. Notably, Samuelson (1984) and Myerson (1985) characterise surplus maximising mechanisms in a setting with lemons problem.

My paper *combines* the static competitive setting the mechanism design approach; there is an intermediary, who coordinates the sale of the object between the seller and the buyer, but there is *also* a Walrasian market in the background, and the seller always has the option to sell here. My main finding when the mechanism operates *alongside* the market, rather than replacing it, then the mechanism's ability to screen may be greatly disrupted. Another difference with Samuelson (1984) and Myerson (1985) is that I use a stronger notion of IR for the buyer; there, the buyer's IR is over the entire trading process, and the only requirement is that his ex ante expected payoff from participating in the mechanism needs to be non negative. I require IR to be satisfied at **every** price: for trade to happen at any price, the buyer should not anticipate a loss from trading at that price. This is the same as the *veto incentive compatibility* requirement in Gerardi, Hörner, & Maestri (2014).

The presence of the market alongside the intermediary also connects my paper to the literature on mechanism design with a "competitive fringe", started by Philippon & Skreta (2012), and Tirole (2012). These papers study optimal government interventions to restore lending and investment in a market with adverse selection, where following government intervention, firms can raise funds in a static, competitive market. Like my paper, the market, and the mechanism offered by the government affect in other in equilibrium. Participation in the government's program signals private information and therefore endogenously affects the market, and the market in turn influences the decision to participate in the government's program. The setting, and the nature of intervention, however, is quite different from mine, and so are some results. In particular, in these papers, the government never benefits from "shutting down" the market, whereas in my setting, under some conditions, the market completely takes away the intermediary's ability to screen, so when these conditions hold, if "shutting down" the market was possible, it would be strictly optimal.

Another strand of literature that this paper is related to is that on *ratifiable mechanisms*, as in Cramton & Palfrey (1995) and Celik & Peters (2011). In these papers, the outside option to the mechanism takes the form of a *game*. Players can either participate in (or "ratify") the mechanism, or reject it. If any player rejects the mechanism, all players play the game. The similarity with my paper is that the act of rejecting the mechanism conveys information about a player's type, and influences other players' beliefs about him when the game is played. The main difference from my work is that in these papers, the choice between the mechanism and the game is made *ex ante*, and if all agents choose the mechanism, they are *bound* to the mechanism. In my setting, the choice of whether to accept the intermediary's mechanism or not, is made at an *interim* stage, once the seller knows whether the option to sell through the intermediary exists or not. Another difference is that unlike these papers, I consider a setting with adverse selection.

My paper is also related to the literature that combines information design and mechanism design; examples include mechanism design with "aftermarkets", as in Dworczak (2020), conflict resolution as in Balzer & Schneider (2019), and the literature on sequential agency by Calzolari & Pavan (2006) and Calzolari & Pavan (2009). Like my paper, the design of the mechanism influences what happens outside the mechanism. The difference is that in these papers, the the mechanism designer can choose to reveal some information elicited from the agent, to influence the *post-mechanism* outcome. In my paper, the intermediary cannot directly reveal any information to the market. It can only influence what the market learns about the seller's type *in equilibrium*, through its choice of menu.

2 A Two type Example: Complete Market Shutdown

A seller has an indivisible good that she'd like to sell. The good's *quality* takes one of two possible values: θ_H, and θ_L, where $\theta_H > \theta_L > 0$, and the probability that the good is of quality θ_H is denoted by $\mu(\theta_H)$. There are two channels through which the seller can sell: an *intermediary* and a *market*. The intermediary has a single buyer associated with it and the market has a large number of potential buyers associated with it; all buyers are identical.[2]

For a good of quality θ, the seller's cost of providing the good is equal to θ, and the buyers' utility is $(1 + \alpha)\theta$, where $\alpha \in (0, 1)$ reflects the gains from trade. The realisation of θ is the seller's private information, and is referred to as her *type*. The distribution however, is common knowledge.

I assume that $(1 + \alpha)\mathbb{E}[\theta] < \theta_H$, where the Expectation is taken with respect to the prior $\mu(.)$. I refer to this as the *lemons condition*; it means that given the prior, the maximum price that a buyer is willing to pay for the good is strictly lower than the cost for type θ_H, which is the minimum price at which a seller of type θ_H would sell.

The intermediary can commit to a menu of allocations, where each allocation in the menu is a tuple (π, p). If the seller chooses allocation (π, p), then with probability π, she will have opportunity to sell through the intermediary at price p. Here, p, is the price *conditional* on sale; the seller only gets it in the event of a sale happening through the intermediary. The market, on the other hand, offers the option to sell at a *single* price p_M. In equilibrium, p_M is determined by the market clearing condition: it is the expected value of the good for the buyers on the market, *conditional* on the good being sold on the market.

The timeline is as follows:

1. The intermediary commits to a menu of allocations.
2. The seller chooses an allocation from this menu.
3. The uncertainty associated with the intermediary is resolved. If the seller chose allocation (π', p'),
    - with probability π', she gets the option to sell through the intermediary.
    - with probability $1 - \pi'$, this option does not exist.
4. The seller decides *where* to sell, if at all.
    - If the seller *has* the option of selling through the intermediary, she decides whether to i) sell at p' through the intermediary, ii) sell at p_M on the market, or iii) not sell at all.
    - If she cannot sell through the intermediary, she decides between i) selling at p_M on the market, and ii) not selling.

[2] I assume that the buyer associated with any channel can only buy through *that* channel.

5. If the seller is selling through the intermediary (resp. the market), the buyer(s) buy as long as *conditional* on sale happening at p' (resp. p_M) the buyer's expected value for the good is at least as much as the price.

Thus, the menu chosen by the intermediary induces a game where the seller chooses an allocation, and where to sell, and then the buyers choose whether or not to buy. I look at Perfect Bayesian Equilibria (PBE) of this game. Observe that I do not specify the objective function of the intermediary; this is because my focus is what is *feasible* in equilibrium for any menu, rather than on which menu is optimal given the intermediary's objective function. In equilibrium, p_M is the expected value of the good for the buyers on the market, where the expectation is taken with respect to the distribution of types that trade on the market in equilibrium. For example, if only the low type sells on the market in equilibrium, then p_M is $(1+\alpha)\theta_L$.

I find that when the intermediary operates alongside the market, there exists *no* equilibrium in which a good of quality θ_H is traded with positive probability. Before getting to this main result, I describe what would happen if either i) only the market existed, or ii) only the intermediary existed. In particular, we will see that if the intermediary is operating in isolation, then we do have an equilibrium in which θ_H trades with positive probability.

Fact 1. *If the market is the only place where the seller can sell, then in equilibrium, θ_H can never sell with positive probability.*

This follows directly from the *lemons condition.* For type θ_H to be willing to sell, p_M has to be at least θ_H. But then, at such a price, type θ_L would also find it strictly optimal to sell. So, conditional on p_M, the buyers' beliefs equal the prior, and so by the *lemons condition*, their expected value for the good is strictly lower than θ_H, and therefore, they wouldn't buy. Thus, the unique equilibrium outcome is that only type θ_L trades in equilibrium, and $p_M = (1+\alpha)\theta_L$.

Fact 2. *If the intermediary is the* ***only*** *channel for sale, then there exits an equilibrium where θ_L sells with probability one, and θ_H sells with probability $\pi_H \in (0,1)$.*

Proof sketch: Here I consider a menu, and argue that if the intermediary is operating in isolation, and offers this menu, then there exists an equilibrium in which the high quality good is traded with positive probability.

So, suppose the seller can only sell through the intermediary, and the intermediary offers a menu with two allocations: $\mathcal{M}^* = \{\mathcal{L}, \mathcal{H}\}$, where , $\mathcal{L} = (1, (1+\alpha)\theta_L)$, $\mathcal{H} = (\pi_H, \theta_H)$, and $\pi_H \in (0,1)$. When this menu is offered, there exists an equilibrium in which on path:

- Type θ_L chooses allocation $\mathcal{L}$ with probability one.
- Type θ_H chooses allocation $\mathcal{H}$ with probability one.
- If the good is being sold at either price $(1+\alpha)\theta_L$, or price θ_H, the buyer buys with probability one.

Given the buyer's strategy, for a seller of type θ, the expected payoff from choosing the allocation $\mathcal{L}$ is $((1+\alpha)\theta_L - \theta)$, and that from choosing $\mathcal{H}$ is $\pi_H(\theta_H - \theta)$. Observe that the *lemons condition* implies that $\theta_H > (1+\alpha)\theta_L$, so type θ_H will always choose allocation $\mathcal{H}$, as the price in allocation $\mathcal{L}$ is strictly lower than θ_H, her cost of providing the good. Type θ_L, however, faces the following trade-off: she can choose allocation $\mathcal{L}$, and sell with probability one at price $(1+\alpha)\theta_L$, or choose $\mathcal{H}$, and sell at a strictly higher price θ_H, but with with probability $\pi_H < 1$. Suppose π_H satisfies the following:

$$\pi_H(\theta_H - \theta_L) = ((1+\alpha)\theta_L - \theta_L)$$

So, type θ_L is indifferent between the allocations $\mathcal{H}$ and $\mathcal{L}$, and therefore, it is indeed sequentially rational for type θ_L to choose $\mathcal{L}$ with probability one.[3] For the buyer, if the good is being sold at price $(1+\alpha)\theta_L$, his equilibrium beliefs are that its type θ_L with probability one, so it is sequentially rational for the him to buy at this price. Similarly, since *only* type θ_H is selling at price θ_H, it is optimal for the buyer to buy at this price too. Therefore, in equilibrium,

- The low quality good is sold with probability one, at price $(1+\alpha)\theta_L$.
- The high quality good is sold with probability π_H, at price θ_H.

I now introduce the market, and see what happens when the intermediary has to operate alongside the market.

Proposition 1. *If the intermediary and market* ***coexist****, then unique equilibrium outcome involves θ_L selling with probability one and θ_H with probability zero.*

The above results says that the market completely disrupts the intermediary's functioning. I first provide some intuition behind the above result, and then use the menu $\mathcal{M}^*$ to illustrate the core logic of the proof.

[3] $\pi_H(\theta_H - \theta_L) = ((1+\alpha)\theta_L - \theta_L)$ implies that $\pi_H = \frac{\alpha\theta_L}{\theta_H - \theta_L}$, which is strictly lower than one by the *lemons condition*.

The intuition behind the result is that for θ_H to trade in equilibrium, it must be *separated* from θ_L. This separation is needed because of the *lemons condition*: suppose, in equilibrium, *both* types are selling at the same price p'. Then p' must be at least θ_H to satisfy the IR for the high type. But since *both* types are selling at p', the buyers' beliefs conditional on this price equal the prior, so by the *lemons condition*, he will not buy at $p' \geqslant \theta_H$.

We also saw that when the intermediary is operating alone, separation is achieved through a lower probability of trade for the high type: in $\mathcal{M}^*$, θ_L is deterred from choosing allocation $\mathcal{H}$ because $\pi_H < 1$. Therefore, in equilibrium, *only* the high type is selling at the higher price θ_H, so the buyer is willing to buy, and high type trades with positive probability in equilibrium. The presence of the market interferes with this screening through allocation probabilities. This is because when the intermediary operates alongside the market, the seller can sell on the market, in case trade through the intermediary does not materialise. Therefore the types can no longer be separated, and θ_H cannot trade in equilibrium. I now illustrate this disruption in more detail:

Proof Sketch: I now provide a sketch of the proof of Proposition 1, by using the menu $\mathcal{M}^*$. Recall that in the absence of the market, if the intermediary offers $\mathcal{M}^*$, then there *is* an equilibrium where the high type trades with positive probability. I will now argue that ***if $\mathcal{M}^*$ is offered when the intermediary is operating alongside the market, there no longer exists an equilibrium in which the high type trades with positive probability.***

Firstly, observe that in any equilibrium, p_M has to be at least $(1+\alpha)\theta_L$, since the distribution of types conditional on the seller selling on the market cannot be worse than degenerate at θ_L. In fact, *it can be shown that in any equilibrium, $p_M = (1+\alpha)\theta_L$.*

Now suppose, by contradiction, that there *is* an equilibrium with menu $\mathcal{M}^*$ in which θ_H trades with positive probability. Fix such an equilibrium. In this equilibrium, it must be that the high type is selling through the intermediary, at price θ_H. This is because the market price p_M, and the other price in $\mathcal{M}^*$, are both equal to $(1+\alpha)\theta_L$, which is strictly lower than θ_H. So, the high type cannot sell on the market or at the other price in the menu.

Therefore, in equilibrium, the buyer's strategy must be to buy, if the good is being sold at price θ_H through the intermediary. Given the buyer's strategy, we can see that *now*, the low type strictly prefers allocation $\mathcal{H}$ to $\mathcal{L}$. This is because by choosing $\mathcal{H}$, with probability $\pi_H > 0$ she can sell at price θ_H, and with probability $1 - \pi_H$, when she doesn't have the option to sell through the intermediary, she can sell on the market at $p_M = (1 + \alpha)\theta_L$. On the other hand, by choosing $\mathcal{L}$, her only option is to sell at price $(1 + \alpha)\theta_L$, either through the intermediary, or on the market. But then, in equilibrium, *both* types will choose the allocation $\mathcal{H}$. So, conditional on the price θ_H, the buyer's beliefs equal the prior, and he will not buy. This contradicts that type θ_H is able to sell with positive probability in this equilibrium.

This reasoning extends more generally; there is **no** menu that the intermediary can offer such that there is an equilibrium with that menu where the high type trades with positive probability. As with menu $\mathcal{M}^*$, the idea is that now, in equilibrium, type θ_L is *guaranteed* a price of $(1 + \alpha)\theta_L$ on the market. This destroys any separation that the intermediary can achieve through allocation probabilities, because now, the lower probability of trade in the allocation *meant* for the high type served is no longer an effective deterrent for the low type to *not* choose this allocation.

3 Model

I now develop a model of bilateral trade with an intermediary and a static competitive market.

3.1 Setup

I consider a bilateral trade setting with one seller and one buyer. The seller has one unit of an indivisible good for sale. This good has a quality θ associated with it, which is drawn from the set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, where $\theta_1 > \theta_2, \ldots > \theta_n > 0$, and θ is drawn according to the distribution $\mu(.)$. The realisation of θ is the seller's private information, but the distribution $\mu(.)$ is common knowledge. I refer to θ as the seller's *type*.

This is a setting with *interdependent values*; for a good of quality θ, the seller's cost of parting with the good is θ, and the buyer's utility from the good is $(1 + \alpha)\theta$, where $\alpha \in (0, 1)$. So, if the seller sells to the buyer at price p, the seller's payoff is $p - \theta$, and the buyer's payoff is $(1 + \alpha)\theta - p$. Since $\theta_n > 0$, this implies that the gains from trade are always strictly positive. The seller and the buyer are risk neutral.

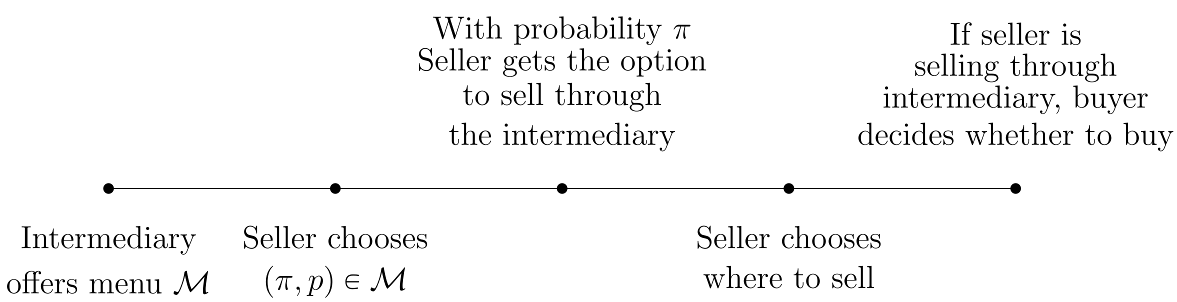
Assumption 1. ***Lemons condition**: The prior $\mu(.)$ satisfies the following condition: $(1 + \alpha)\mathbb{E}[\theta] < \theta_1$.*

The above condition says that adverse selection is *severe*; given the prior, the maximum price that the buyer would be willing to pay for the good is strictly less than the cost of providing the good for the highest type.

There are two channels through which the seller can sell her good – an *intermediary* and a *static competitive market* ("market" henceforth). I assume that the buyer is not mobile, and though the seller can sell anywhere, a buyer can only buy through the intermediary. Moreover, I do not model the buyer(s) on the market explicitly. Rather, I assume there is a large number of potential buyers associated with the market, all identical to the buyer associated with the intermediary, and they can only purchase on the market.[4] Henceforth, "buyer" is used to refer to the buyer associated with the intermediary, unless I specify that we are taking about the buyer(s) on the market.

The intermediary offers a finite menu of allocations $\mathcal{M} \subseteq [0, 1] \times \mathbb{R}_+$ to the seller. An element of the menu $\mathcal{M}$ is denoted by (π, p); if the seller chooses this allocation, with probability π, she will have the option to sell her good through the intermediary, at price p. Here, p is the price *conditional* on sale, the seller gets it only if a sale actually happens. The market has a single price p_M associated with it, where p_M is determined in equilibrium. Without loss, I assume that the menu offered by the intermediary must always contain the allocation $(0, 0)$; this corresponds to the seller deciding to not sell through the intermediary.

3.2 Timeline



The menu chosen by the intermediary induces a game where the seller first decides which allocation to choose and then where to sell, and then, if the seller is selling through the intermediary, the buyer decides whether or not to buy (if the seller is selling on the market, the good always gets sold). look at the Perfect Bayesian Equilibria (PBE) of this game. In equilibrium, the market price p_M is determined by the market clearing condition; it is equal to the expected value of the good for the buyers on the market, where the

[4] I make this assumption that the buyers are not mobile across channels to abstract away from the buyers' choice of where to trade, as incorporating this aspect would not only make the problem more complicated, but would also distract from my main focus

expectation is taken with respect to the distribution of the seller's types that sell on the market in equilibrium. For example, if *only* type θ sells on the market in equilibrium, then $p_M = (1+\alpha)\theta$. I define the strategies, and equilibrium formally in the next subsection.

3.3 Strategies

Fix a menu $\mathcal{M}$ offered by the intermediary. I now define the strategies of the seller and buyer formally. I also define the buyer's beliefs and the beliefs on the market.

Seller's strategy: The seller's strategy has multiple components. It specifies i) the allocation that she chooses, as a function of her type, and ii) where she decides to sell, given her choice of allocation, her type, and whether the option to sell through the intermediary exists. The seller's strategy is given by $\sigma_S = (\sigma(.), \gamma_I(.), \gamma_M(.), \gamma_M'(.))$. I now describe each component individually.

The first component describes the seller's choice of allocation in $\mathcal{M}$, as a function of her type:

$$\sigma : \Theta \to \Delta\mathcal{M}$$

Here, $\sigma((\pi,p)|\theta)$ denotes the probability with which type θ chooses the allocation $(\pi,p) \in \mathcal{M}$. For any choice of allocation, the opportunity to trade through the intermediary may or may not realise. If the seller chooses an allocation, and *gets* the option to sell through the intermediary, she can either choose to sell through the intermediary, or on the market, or not sell at all. This choice is captured by the following two functions:

$$\gamma_I, \gamma_M : \Theta \times \mathcal{M} \to [0,1]$$

Suppose the seller's type is θ, she chose allocation (π,p) in the menu, *and* has the option to put the good up for sale through the intermediary at price p. Then, $\gamma_I((\pi,p),\theta)$ denotes the probability with which she chooses to sell through the intermediary, at price p, and $\gamma_M((\pi,p),\theta)$ denotes the probability with which she chooses to sell on the market. Therefore, for any θ, and any (π,p), $\gamma_I(\theta,(\pi,p)) + \gamma_M((\pi,p),\theta) \leqslant 1$, where $1 - \gamma_I(\theta,(\pi,p)) - \gamma_M((\pi,p),\theta)$ is the probability with which the seller does not sell.

Lastly, the seller may not get the opportunity to sell through the intermediary. In this case, she can either decide to sell on the market, or not at all. This is the last component of the seller's strategy:

$$\gamma_M^{'} : \Theta \times \mathcal{M} \to [0,1]$$

Here, $\gamma_M^{'}((\pi,p),\theta)$ denotes the probability with which type θ chooses to sell on the market conditional on choosing (π,p) in the menu, and *not* having the option to sell through the intermediary. With probability $1-\gamma_M^{'}((\pi,p),\theta)$, she chooses not to sell at all.

Buyer's strategy: The buyer's strategy describes whether or not the buyer buys at any price p, if the seller is selling through the intermediary at this price. It is a function $\sigma_B : \mathbb{R}_+ \to \{0,1\}$, where $\sigma_B(p) = 1$ denotes that the buyer buys at price p. Observe that I rule out randomisation by the buyer, and restrict the buyer to pure strategies.[5]

I do not specify the strategies of the of the buyers on the market explicitly. There are a large number of potential buyers on the market and in any equilibrium, and in any equilibrium, the market price p_M is determined by the market clearing condition. So, the seller is always able to sell on the market at p_M with probability one, if she decides to do so.

Given the buyer's strategy σ_B, the maximum expected payoff for a seller of type θ, if she chooses an allocation (π,p) in $\mathcal{M}$ is given by:

$$\max_{\gamma_I,\gamma_M,\gamma_M^{'}\in[0,1]} \quad \pi(\gamma_I p \mathbb{1}_{\sigma_B(p)=1} + \gamma_M p_M) + (1-\pi)\gamma_M^{'} p_M \tag{1}$$

I denote the expression in 1 as $V((\pi,p),\theta|\sigma_B,p_M)$. Here, γ_I, γ_M, and $\gamma_M^{'}$ are shorthand for $\gamma_I((\pi,p),\theta)$, $\gamma_M((\pi,p),\theta)$, and $\gamma_M^{'}((\pi,p),\theta)$ respectively. $\mathbb{1}_{\sigma_B(p)=1}$ is the indicator function that denotes whether the buyer buys or not, if the seller decides to sell at p. Observe that it is without loss that the seller always chooses *some* allocation in the menu, because choosing to sell directly on the market can be captured by choosing the allocation $(0,0)$ in the menu.

Beliefs: I use $\mu_B : \mathbb{R}_+ \to \Delta\Theta$ to denote the buyer's beliefs about the seller's type as a function of the type, and for any p' such that $(\pi',p') \in \mathcal{M}$, $\mathbb{E}_B[\theta|p']$ denotes the expected value of the seller's type, *conditional* on the seller selling at price p', where the expectation is taken with respect to $\mu_B(p')$. I denote the beliefs on the market by $\mu_M \in \Delta(\Theta)$, this represents the beliefs of all buyers on the market about the seller's type, conditional on

[5]Restricting the buyer to pure strategies is actually without loss, as any randomisation that the buyer might do can be built into the allocation probabilities in the menu offered by the intermediary. So, for (π,p) if the buyer randomises between buying and not buying at p, then the same equilibrium outcome is attainable by lowering π, and adjusting the buyer's strategy so that he buys with probability one.

the seller deciding to sell on the market.

Assumption 2. *For any p, $\sigma_B(p) = 1$ if and only if $(1+\alpha)\mathbb{E}_B[\theta|p] \geqslant p$.*

So, I assume that at any price, the buyer buys, as long as given his beliefs about the seller's type conditional on the price, he *does not anticipate a loss* from buying. So he buys when indifferent.[6] I refer to this as the buyer's ***interim IR*** condition.

3.4 Equilibrium and Payoffs

The solution concept is Perfect Bayesian Equilibrium. Fix any menu $\mathcal{M}$. An equilibrium of the game induced by the menu is given by $(\sigma_S, \sigma_B, \mu_B, \mu_M, p_M)$, where

- Given σ_B and p_M, for any type θ, the seller's choice of which allocation to choose, and where to sell is sequentially rational. So, $\sigma((\pi,p)|\theta) > 0$ if and only if $V((\pi,p),\theta|\sigma_B,p_M) \geqslant V((\pi',p'),\theta|\sigma_B,p_M)$ for all $(\pi',p') \in \mathcal{M}$. Following any choice of allocation, the choice of if, and where to sell must be sequentially rational, given σ_B, and p_M.[7]

- Given μ_B, the buyer's choice of buying at any price p is sequentially rational; buyer buys if and only if the buyer's interim IR at that price is satisfied, i.e., $(1+\alpha)\mathbb{E}_B[\theta|p] \geqslant p$.

- Given the seller's strategy σ_S, μ_B is derived using Bayes Rule wherever possible.

- $p_M = (1+\alpha)\mathbb{E}_M[\theta]$, where the $\mathbb{E}_M$ denotes the Expectation taken with respect to $\mu_M(.)$, the equilibrium beliefs of the buyers on the market about the types of the seller that sell on the market, where $\mu_M(.)$ is derived from the seller's strategy using Bayes Rule, whenever trade takes place on the market with positive probability in equilibrium.[8] So, when trade takes place with positive probability on the market, $\mu_M(.)$ represents the equilibrium distribution of the types of the seller that sell on the market.

[6] This is again without loss. See footnote 5.

[7] The choice of if, and where to sell must be sequentially rational even if the choice of allocation is *off-path*, i.e., $\sigma((\pi,p)|\theta) = 0$.

[8] When trade takes place on the market with positive probability, all buyers on the market have the same beliefs, but I assume that even when trade takes place on the market with probability zero, all buyers on the market still have the same off path beliefs about the seller's type.

Outcomes and Payoffs: For any menu $\mathcal{M}$, let $P_{\mathcal{M}} = \{p|(\pi, p) \in \mathcal{M}\}$; this is the set of all the prices at which sale can possibly take place through the intermediary. An *outcome* of the game corresponding to $\mathcal{M}$ is a tuple (θ, i, p'), where $i \in \{I, M\}$. The tuple represents the outcome that the good of type θ was sold on i at price p', where i can be either through the intermediary (I), or the market (M). If $i = M$, then p' must be p_M. For a seller of type θ, his payoff from the outcome (θ, i, p') is $p' - \theta$, and the payoff for the buyer who purchased the good is $(1 + \alpha)\theta - p$.

An equilibrium induces, for any type, a probability distribution over outcomes, which is represented by $(\{\pi_p(\theta)\}_{p\in P_{\mathcal{M}}}, \pi_M(\theta))$, where $\pi_p(\theta)$ is the equilibrium probability that the good of type θ is sold through the intermediary at price p, and $\pi_M(\theta))$ is the probability that this good is sold on the market at p_M. For any θ, It must be that $\sum\{\pi_p(\theta)\}_{p\in P_{\mathcal{M}}} + \pi_M(\theta)) \leqslant 1$, where, if this sum is strictly less than 1, then this means that with some probability, type θ does not sell in this equilibrium. The seller and the buyers are risk neutral; the seller's expected payoff from $(\{\pi_p(\theta)\}_{p\in P_{\mathcal{M}}}, \pi_M(\theta))$ is $\sum_{p\in P_{\mathcal{M}}} \pi_p(\theta)(p - \theta) + \pi_M(p_M - \theta)$. For the buyer, at the time of buying, this expectation is taken with respect to μ_B, her equilibrium beliefs about the seller's type. On the market, by definition of p_M, any buyer who buys gets zero payoff.

Menus vs Direct Mechanisms: In my model, the intermediary can commit to menus, which are indirect mechanisms. I do not consider direct mechanisms that map reports in Θ to tuples (π, p), because direct mechanisms are *not* without loss here. This is because I am implicitly restricting attention to a special class of *deterministic* mechanisms: although the menus are stochastic in the sense that at any price, trade may or may not happen, each allocation in the menu consists of a *single* price, so the intermediary is not offering a menu of *lotteries* over prices.[9] As Strausz (2003) shows, when restricting attention to deterministic mechanisms, direct IC mechanisms may *not* be without loss. The main idea is that in the game *induced* by the mechanism, there are outcomes which are only attainable when the seller plays a mixed strategy. A mechanism that offers lotteries over prices can randomise *for* the seller, but my class of mechanisms cannot, so the standard Revelation Principle does not hold.[10] Section 4.5 contains a more in-depth discussion of what happens when the intermediary can offer lotteries over prices.

[9] I restrict attention to this class of mechanisms for tractability, but as I discuss in Section 4.5, the breakdown result with two types still goes through, even if I allow for lotteries over prices.

[10] One might wonder why this randomisation over prices can make a difference, since the seller and buyer are risk neutral. The reason is that in my model, the buyer's IR must hold for *every* price, and therefore randomisation over prices can help by allowing more flexibility in varying the information contained in any *given* price.

4 Main Results

With two-types, the market completely disrupts the operation of the intermediary. In this section, I study the market's effect on the intermediary's functioning more generally. But, before getting to the main results, I first describe how the intermediary screens when there is no market, and what changes when the intermediary is operating alongside the market. This will be useful for understanding the challenges to screening when the intermediary has to operate alongside the market.

4.1 Screening When There Is No Market

When the intermediary is operating in isolation, it screens through a trade-off between prices and probability of trade. In the intermediary's menu, allocations that have higher price have lower probabilities of trade.

Why does this trade off cause separation of higher and lower types in equilibrium? The key idea is that higher types have a higher cost of parting with the good. So, while comparing two allocations, they might find allocation with a lower probability of trade more attractive, because it involves a lower expected cost of parting with the good. In equilibrium, this results in higher types choosing allocations with higher prices and lower probability of trade.

To see this more clearly, observe that for any allocation (π, p), and any price θ, the expected payoff from choosing this allocation is $\pi(p - \theta) = \pi p - \pi\theta$, where πp is the *expected price from sale* in this allocation, and $\pi\theta$ is the *expected cost of parting with the good*. Now, consider allocations (π, p) and (π', p'), where $\pi < \pi'$, and $p > p'$. So, allocation (π', p') has a higher probability of trade, and a lower price. If type θ is comparing the two allocations, then:

$$\pi'(p' - \theta) \geqslant \pi(p - \theta) \iff \pi' p' - \pi p \geqslant (\pi' - \pi)\theta \tag{2}$$

Therefore, the comparison between the allocations boils down to a comparison between $\pi' p' - \pi p$, and $(\pi' - \pi)\theta$. Here, $\pi' p' - \pi p$ is the difference between the expected prices in the two allocations, and $(\pi' - \pi)\theta$ is the difference between the expected cost of parting with the good. Suppose $\pi' p' - \pi p > 0$, so (π', p'), the allocation with the higher probability of sale and lower price, has higher *expected* price from sale than (π, p). But since $(\pi' - \pi)\theta > 0$, (π', p'), *also* involves higher expected cost of parting with the good.

From 2, we can see that for lower values of θ, the higher expected price dominates the higher expected cost, and they prefer (π', p') to (π, p). On the other hand, for higher values of θ, the effect of higher expected cost of parting with the good dominates, and they prefer (π, p). I now sum up this discussion in the following lemma:

Lemma 1. *If type θ is indifferent between allocations (π, p) and (π', p'), where $\pi < \pi'$, and $p > p'$, then any $\theta^{'} < \theta$ strictly prefers (π', p') to (π, p), and any $\theta^{''} > \theta$ strictly prefers (π, p) to (π', p').*

Proof. Follows directly from 2 □

4.2 Screening In The Presence of the Market

In this section, I describe how the market impacts the way the intermediary can screen. The trade off that enables screening remains the same: in equilibrium, higher types trade with lower probabilities, and at higher prices. However, the presence of the market implies certain constraints for the prices at which trade can happen through the intermediary in equilibrium. It also endogenously alters the reservation utility for certain types in equilibrium, thereby changing the way these types evaluate allocations in the intermediary's menu. I now state some lemmas about these equilibrium constraints that will be useful in understanding subsequent results.

Lemma 2. *In equilibrium, if the market price is p_M, then any trade that takes place through the intermediary must be at a price (weakly) greater than p_M. Moreover, in equilibrium, if **any** trade takes place through the intermediary, at a price strictly greater than p_M, then **all** trade through the intermediary must take place at a place at a price strictly greater than p_M.*

Proof. The first part is straightforward. Since, the seller can always sell on the market at p_M, in equilibrium, no type of the seller would sell through the intermediary at $p < p_M$.

For the second part, since there exists a p at which trade happens with positive probability in equilibrium, there must be an allocation (π, p), such that $p > p_M$, and $\sigma_B(p) = 1$, i.e., the buyer's strategy is to buy at p. So, by choosing (π, p), with probability π, the seller can sell at $p > p_M$ (and with $(1 - \pi)$, sell at p_M on the market). So, no type of the seller would choose an allocation (π', p') with $p' = p_M$. □

Lemma 2 is at the root of the breakdown result in the next section. The key idea is that now, types greater than p_M trading in equilibrium has an additional "cost" that was not present when there was no market: it means that all types less than p_M must *also* trade at prices strictly greater than p_M. This, combined with the fact that at any price, the buyer's interim IR constraint must also be satisfied, makes harder for higher types to trade in equilibrium.

I now argue that for types $\theta \leqslant p_M$, the presence of the market alters their reservation utility, while evaluating allocations in the menu.

Lemma 3. *Suppose the equilibrium market price is p_M. Then any two types, (weakly) lower than p_M, have the same "effective" reservation utility, and therefore have the same ranking over any two allocations.*

Proof. Consider allocations (π, p) and (π', p') in $\mathcal{M}$, such that $\pi < \pi'$, $p > p' > p_M$, and in equilibrium, the buyer' strategy is to buy at both prices p and p'. Then, for any $\theta \leqslant p_M$, the payoff from choosing (π, p) is

$$\pi(p - \theta) + (1 - \pi)(p_M - \theta) = \pi(p - p_M) + p_M - \theta$$

Similarly, the payoff from choosing (π', p') is $\pi'(p' - p_M) + p_M - \theta$. Therefore, for θ, the comparison between allocations (π, p) and (π', p') boils down to the comparison between $\pi(p - p_M)$ and $\pi'(p' - p_M)$. Exactly the same thing is true for any $\theta' \leqslant p_M$. So, all types lower than p_M have the same ranking over any two allocations.

□

Therefore, all $\theta \leqslant p_M$ evaluate choices in the menu *as if* their type is p_M, and the intermediary is operating in isolation. I refer to p_M as the *effective type* for all types $\theta \leqslant p_M$. I now use this fact to prove the following lemma:

Lemma 4. *Suppose, in equilibrium, an allocation (π, p) is chosen by some $\theta \leqslant p_M$, and by some $\theta' > p_M$. Then, in equilibrium, is is also chosen by $\{\theta | p_M < \theta < \theta'\}$.*

Proof. This follows directly from Lemma 1 and the notion of *effective type*. Fix any θ such that $p_M < \theta < \theta'$ (if such a θ exists). Then, θ strictly prefers (π, p) to all allocations (π', p') such that $\pi' > \pi$, and $p' < p$. This is because *effective type* p_M chooses, and therefore weakly prefers (π, p) to (π, p). Therefore, by Lemma 1, since $\theta > p_M$, θ strictly prefers (π, p) to (π, p). Similarly, we can argue that θ will not choose any allocation (π'', p'') such that $\pi'' < \pi$ and $p'' > p$, by using θ'. □

4.3 Main Result: Market Breakdown

With two-types, the presence of a market leads to a trading impasse. This impasse result holds more generally; *with finitely many types, under some conditions on the prior,* *only the lowest type trades in any equilibrium.* Recall that the set of types is given by $\Theta = \{\theta_1, \theta_2, \dots \theta_n\}$, where $\theta_1 > \theta_2 \dots > \theta_n$, and the probability of type θ is $\mu(\theta)$.

Definition 1. ***Bottom Lemons Condition***: *The prior $\mu(.)$ satisfies the Bottom Lemons Condition (BLC) if for any $k \in \{1, 2 \dots n-1\}$, we have that $(1+\alpha)\mathbb{E}[\theta' | \theta' \leqslant \theta_k] < \theta_k$.*

When there are two types, BLC is *equivalent* to the *lemons condition.* With more than two types, this condition says that for *any* subset of types at the bottom, there is a *lemons problem.* Before stating the main result, I provide two examples to help understand the BLC better.

Suppose there are three possible types: $\theta_1 = 3$, $\theta_2 = 2$, and $\theta_3 = 1$, and $\alpha = 0.2$. So the buyer's payoff from a good of type θ is 1.2θ. Keeping α and the set of possible types same, I vary the prior to provide two examples: one where the prior satisfies the BLC, and another where it does not.

Example 1. *Prior satisfies BLC: $\mu(1) = \mu(2) = \mu(3) = \frac{1}{3}$.* First consider the two lower types: conditional on $\theta \in \{\theta_2, \theta_3\}$, the buyer's expected value for the good is $(1+\alpha)\mathbb{E}[\theta | \theta \leqslant \theta_2] = (1.2)(\frac{2+1}{2}) = 1.8$ which is strictly lower than θ_2. Now, consider the entire set of types. The buyer's expected value, given the prior, is $(1+\alpha)\mathbb{E}[\theta] = 2.4$, which is strictly lower than θ_1.

Example 2. *Prior does **not** satisfy BLC: $\mu(2) = \frac{3}{5}$, and $\mu(1) = \mu(3) = \frac{1}{5}$.* Now, $(1+\alpha)\mathbb{E}[\theta | \theta \leqslant \theta_2] = 2.8 > \theta_2$, so the BLC fails, since conditional on the lower two types, the buyer's expected value is strictly higher than θ_2. The *lemons condition* still holds though, as $(1+\alpha)\mathbb{E}[\theta] = 2.4 < \theta_1$. So, the prior violates the BLC, but still satisfies the overall *lemons condition.*

I now state the main result, which says that BLC characterises conditions under which total breakdown of trade is the *unique* equilibrium outcome.

Theorem 1. *The unique equilibrium outcome involves type θ_n trading with probability one, and all $\theta > \theta_n$ trading with probability zero if and only if the BLC is satisfied.*

Proof sketch: Before getting to the sketch of the proof, I make the following observation: *BLC implies that types at the bottom cannot be "pooled".* To see this, suppose there is such pooling in equilibrium, i.e., there exists a $\overline{\theta} > \theta_n$, such that all $\theta \leqslant \overline{\theta}$ trade at the same price p' in equilibrium. Then, conditional on p', the buyer's expected value for the good is $\mathbb{E}[\theta|\theta \leqslant \overline{\theta}]$, which, by BLC, is *strictly* lower than $\overline{\theta}$. But since $\overline{\theta}$ is selling at p' in equilibrium, we must have $p' \geqslant \overline{\theta}$. This is a contradiction.

I will argue that any equilibrium where types $\theta > \theta_n$ trade, *must* have such pooling, thereby resulting in a contradiction. The key force behind the inevitability of pooling is that when the market is present, the lowest type can never be deterred from mimicking some higher type's choice of allocation, and everything unravels from here. I now go through the sketch of the proof in a series steps. The full proof is in the Appendix.

*Step 1: In any equilibrium, the price on the market is $p_M = (1+\alpha)\theta_n$, and **only** θ_n can trade on the market.* I do not provide a proof of why $p_M = (1+\alpha)\theta_n$ here, but I prove the second part taking this as given. BLC implies that $(1+\alpha)\theta_n$ is strictly lower than θ_{n-1}, the second-lowest type. Therefore, $(1+\alpha)\theta_n$ is strictly lower than any type greater than θ_n. Since $p_M = (1+\alpha)\theta_n$ in any equilibrium, only θ_n can trade on the market. And for a seller of type $\theta > \theta_n$, since the market price is lower than her reservation utility, if she trades in equilibrium, it must be through the intermediary.

Now suppose there is an equilibrium in which types greater than θ_n trade with positive probability. Fix such an equilibrium. Then, the following is true:

*Step 2: In equilibrium, θ_n mimics the allocation choice of **some** higher type.* Suppose not, i.e., in equilibrium, θ_n chooses an allocation that's *not* chosen by *any* $\theta > \theta_n$. Let this allocation be (π', p'). Since, in equilibrium, *only* θ_n is choosing this allocation, therefore the buyer only finds it optimal to buy if $p' \leqslant (1+\alpha)\theta_n$.

But it cannot be optimal for θ_n to choose an allocation with $p' \leqslant (1+\alpha)\theta_n$ in equilibrium. Recall that $\theta > \theta_n$ trade with positive probability in equilibrium, and these types can only trade through the intermediary. So, if they trade with positive probability in equilibrium, there exists an allocation (π, p), where $p \geqslant \theta_{n-1} > (1+\alpha)\theta_n$, such that by choosing this allocation, the seller can sell at p with probability $\pi > 0$. This is a contradiction. Therefore, in equilibrium, θ_n chooses an allocation that's *also* chosen by some $\theta > \theta_n$.

*Step 3: Step 2 implies that there must be **pooling at the bottom**; there exists a type $\overline{\theta} > \theta_n$, such that **all** $\theta \leqslant \overline{\theta}$ choose the **same allocation** in equilibrium.* Let the allocation chosen by θ_n in equilibrium be (π^*, p^*). By *Step 2*, there exists a $\theta^{'} > \theta_n$, such that in equilibrium, $\theta^{'}$ chooses (π^*, p^*) as well. Let $\overline{\theta}$ be the highest θ that chooses (π^*, p^*) in equilibrium. Since both $\theta_n < p_M$, and $\overline{\theta} > p_M$ choose (π^*, p^*) in equilibrium, by Lemma 4, we have that all types in the set $\{\theta | p_M < \theta < \overline{\theta}\}$ (if any), must *also* choose (π^*, p^*) in equilibrium. Therefore, the set of types that chooses (π^*, p^*) in equilibrium, is given by $\{\theta | \theta \leqslant \overline{\theta}\}$.

Step 4: Step 3 results in a ***contradiction****.* By *Step 3*, there exists a $\overline{\theta} > \theta_n$, such that all $\theta \leqslant \overline{\theta}$ choose the same allocation in equilibrium. Let this allocation be (π^*, p^*), so the buyer's expected value for the good, conditional on price p^*, is $(1 + \alpha)\mathbb{E}[\theta | \theta \leqslant \overline{\theta}]$. But by BLC, $(1 + \alpha)\mathbb{E}[\theta | \theta \leqslant \overline{\theta}] < \overline{\theta}$, so the IR of $\overline{\theta}$ is violated at p^*, which contradicts that $\overline{\theta}$ chooses (π^*, p^*) in equilibrium.

This completes the sketch of the proof. If we start with an equilibrium where types greater than θ_n trade with positive probability, we reach a contradiction, so there can be no such equilibrium. I only sketched the proof for sufficiency: the result in Theorem 1 is an *if and only if*, so if the BLC is *not* satisfied, then there always exists an equilibrium in which types greater than θ_n trade with positive probability. I give an example of such an equilibrium in the next subsection.

When BLC is satisfied, we can do *strictly* better in terms of surplus from trade when the intermediary is operating in isolation. One trivial construction that allows this is the exactly menu that I constructed for the two type case; the menu offers exactly two allocations and only the lowest two types, θ_n and θ_{n-1} trade with positive probability in equilibrium. θ_n trades with probability one at price $(1 + \alpha)\theta_n$, and θ_{n-1} with probability $\pi_{n-1} < 1$ at price θ_{n-1}. Of course, this might not be the surplus maximising menu, and we can have menus such that $\theta > \theta_{n-1}$ also trade in equilibrium. The main point is if there is no market, we can *always* do better than *just* θ_n trading with probability one.

Observe that the BLC characterises the condition on the prior under which, if there is no intermediary, and the static competitive market is the only channel for sale, then the unique equilibrium outcome is that only θ_n trades in equilibrium. So, the result in Theorem 1 says that if adverse selection is "severe" enough that in the absence of the intermediary, the market features a total breakdown of trade, then the intermediary is *also* completely ineffective in averting this breakdown when it has to operate *alongside* the market.

4.4 What if the BLC fails?

In this section, I describe what happens when the BLC is violated. I first show that in this case, we can always avoid breakdown. Then, I show that although a complete breakdown of trade can be avoided, the the presence of the market might still result in some inefficiency. Finally, I state that when the BLC is violated, the presence of the market can sometimes *improve* efficiency as well.

Fact 3. *When the the BLC is not satisfied, there is always an equilibrium in which types other than the lowest type trade with positive probability.*

Sketch of Proof: The proof is in the Appendix, under the proof of necessity in Theorem 1. Here, I illustrate how we can construct an equilibrium without breakdown through a three-type example. Suppose there are three possible types, so $\Theta = \{\theta_1, \theta_2, \theta_3\}$, where $\theta_1 > \theta_2 > \theta_3$. The prior $\mu(.)$ satisfies the *lemons condition*, so $(1+\alpha)\mathbb{E}[\theta] < \theta_1$. But $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2] > \theta_2$[11], so it does **not** satisfy the BLC. Suppose also that $(1+\alpha)\theta_3 > \theta_2$.[12] Then there exists an equilibrium where 1) trade *only* happens through the intermediary, 2) θ_2 and θ_3 trade with probability one, and 3) θ_1 trades with a positive probability that's strictly lower than one.

I now construct such an equilibrium. Suppose the intermediary offers menu $\mathcal{M} = \{\mathcal{L}', \mathcal{H}'\}$[13], where $\mathcal{L}' = (1, (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2])$, and $\mathcal{H}' = (\pi_H', \theta_1)$, where $\pi_H' \in (0,1)$. BLC implies that $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2] < \theta_1$, so allocation $\mathcal{L}'$ offers the chance to sell at a lower price ($(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$) with probability one, and allocation $\mathcal{H}'$ offers the chance to sell at a higher price (θ_1), but with a probability strictly lower than one. Suppose that π_H' satisfies the following:

$$(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2] > \pi_H'\theta_1 + (1-\pi_H')(1+\alpha)\theta_3 \tag{3}$$

Then, with menu $\mathcal{M} = \{\mathcal{L}', \mathcal{H}'\}$, there is an equilibrium in which $p_M = (1+\alpha)\theta_3$, and on path:

- No trade takes place on the market.

[11] Technically, a violation of BLC in a three type example is equivalent to $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2] \geqslant \theta_2$, so $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$ can be equal to θ_2 as well. In the Appendix, I do not assume this strict inequality while constructing an example without breakdown. I assume this here just to illustrate through a simple example how the intermediary can induce a separating equilibrium, where $\theta > \theta_n$ trade, when the BLC is violated.

[12] Again, this is an additional assumption, and is just for the purpose of this example. The general proof in the Appendix does not make such an assumption.

[13] Technically, $(0,0)$ is also in the menu, but I don't mention it explicitly here.

- Types θ_2 and θ_3 choose $\mathcal{L}'$, and trade with probability one at price $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$.
- Type θ_1 chooses $\mathcal{H}'$, trades with probability $\pi_H' \in (0,1)$, at price θ_1, and does not sell on the market if she is unable to sell through the intermediary.
- The buyer buys at both prices $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$, and θ_1.

To see why there is an equilibrium with the above on path behaviour, I first argue that given p_M, and the buyer's strategy, the choice of allocation of each type of the seller is sequentially rational. Since the *lemons condition* implies that $\theta_1 > (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$, type θ_1 will not choose $\mathcal{L}'$, and will choose $\mathcal{H}'$. Since the *lemons condition also* implies that $\theta_1 > p_M = (1+\alpha)\theta_3$, therefore, if type θ_1 does **not** sell on the market when she is unable to sell through the intermediary.

Now since $\theta_2 < (1+\alpha)\theta_3$, therefore both θ_2 and θ_3 can sell on the market, in case they choose $\mathcal{H}'$, and are unable to sell through the intermediary. But 3 implies that both θ_2 and θ_3 prefer to choose $\mathcal{L}'$, and sell at price $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$ with probability one, as opposed to choosing $\mathcal{H}'$, selling at price θ_1 with probability π_H', and selling on the market with the residual probability.

Now I argue that given the seller's strategy, the buyer's strategy of buying at both prices is sequentially rational. In equilibrium, given that both θ_2, and θ_3 are selling at price $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta_2]$, the buyer indeed finds it optimal to buy. Similarly, since *only* the highest type is selling at price θ_1, the buyer again finds it optimal to buy. I already argued that given the buyer's strategy and $p_M = (1+\alpha)\theta_3$, the optimal strategy for each type of the seller involves not selling on the market. p_M is therefore determined by off path beliefs that if the seller sells on the market, she must be of type θ_3. This concludes the sketch of the proof.

So, we saw that breakdown can be avoided when the BLC does not hold. Can the presence of the market still result in inefficiency? Yes! I provide a sufficient condition on parameters under which the presence of the market reduces the surplus attainable in equilibrium, as compared to when the intermediary is operating in isolation. I first state the result informally:

> **Theorem**: *If a subset of types is "concentrated" at the bottom, such that the highest type in this subset is "sufficiently" lower than all types* ***not*** *in the subset, then the presence of the market results in loss of efficiency.*

Before stating the formal result, I establish some notation.

Definition 2. ***Pooling Type***: *A type θ is said to be a Pooling Type if $(1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant \theta] \geqslant \theta$. Let Θ_{Pool} denote the set of all such types.*

A type θ is *Pooling Type* is such that if the intermediary is operating in isolation, we can find a price p such that in equilibrium, if all types $\theta' \leqslant \theta$ are selling at p with probability one, then both both the buyer's interim IR, and the seller's IR are satisfied. This is because $(1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant \theta] \geqslant \theta$, so we can choose $p \geqslant \theta$, and $\leqslant (1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant \theta]$.

Let $\tilde{\theta} = \max\{\theta|\theta \in \Theta_{Pool}\}$. This denotes the *Highest Pooling Type*.

Theorem 2. *For every Θ, $\mu(.)$, and α, there exists an $\epsilon(\alpha, \mu(.)) > 0$, such that if $\tilde{\theta} > \theta_n$, $\tilde{\theta} < (1+\alpha)\theta_n$, and $\tilde{\theta} - \theta_n < \epsilon(\alpha, \mu(.))$, then, in the surplus maximising equilibrium with a market, i) there is no breakdown, and ii) the expected surplus from trade is strictly lower than the expected surplus in the surplus maximising equilibrium when there is no market.*

By definition of $\tilde{\theta}$, if $\tilde{\theta} > \theta_n$, the the BLC is *not* satisfied. The condition in Theorem 2 says that if $\tilde{\theta} > \theta_n$ *and* sufficiently close to θ_n, i.e. if types at the bottom are *sufficiently* concentrated, then the market leads to loss of efficiency. As in the breakdown case, the main idea behind the inefficiency here is that the market creates an endogenous outside option, and therefore an endogenous IR constraint for the lower types, and increases the minimum price that they would accept in equilibrium. In any equilibrium, $p_M \geqslant (1+\alpha)\theta_n$. Therefore, in any equilibrium, for any $\theta \leqslant \tilde{\theta}$, the *effective type* type is p_M. Since $\tilde{\theta} < (1+\alpha)\theta_n$, we have that for all $\theta \leqslant \tilde{\theta}$, their new , "equilibrium" reservation utility is strictly higher than their original reservation utility. The lower types are precisely the types responsible for the *lemons* problem, so their *effective type* increasing means the gap between the seller's IR and the buyer's interim IR widens, resulting in loss of efficiency.

The final question one might have is that when the BLC is *not* satisfied, can the presence of the market can ever ***improve*** efficiency, compared to when the intermediary is operating in isolation?[14] The answer to this question is yes as well! This may seem counter-intuitive at first; in this setting the screening device is the probability of trade, and it seems natural that the presence of another market where the seller can always sell should reduce the intermediary's ability to screen. Indeed, this is what we have seen till now.

The reason for this is that the intermediary has limited commitment power in the sense that it cannot offer a menu of *lotteries* over prices; each allocation in the intermediary's

[14] When the BLC *is* satisfied, as I showed in Section 4.3, the market always results in a strict reduction in surplus.

menu has exactly *one* price associated with it. In Section 4.5, I discuss why this is a limitation, and why randomisation over prices might help improve surplus in equilibrium, when the intermediary is operating in isolation. The idea behind the market improving surplus is exactly this: it helps with such randomisation, when the intermediary *itself* cannot randomise. I do not go into the details of this here, in Section 6.7 of the Appendix, I provide a three type example for when the market can improve surplus, as well as a detailed discussion about how it improves surplus. Here, I state the result informally.

> **Proposition**: *When the BLC is not satisfied, the presence of the market can sometimes improve surplus, compared to when the intermediary is operating in isolation.*

To sum up, when the BLC is not satisfied, two things can happen. Either the presence of the market reduces surplus, compared to when the intermediary operates alone; Theorem 2 provides a sufficient condition for this to happen. The second possibility is that the presence of the market improves surplus, compared to when the intermediary is operating alone. Section 6.7 of the Appendix contains an example of this.[15]

4.5 What if the Intermediary Could Offer a Lottery Over Prices?

I specified the mechanism offered by the intermediary as a menu, where each allocation in the menu consists of a *single* price. In this section, I first point out that this specification of the intermediary's mechanism is *with* loss, and then talk about how much of my analysis still holds, and which results survive, if I consider more general mechanisms.

Recall that in my model, the buyer's IR must be satisfied at *every* price at which trade happens through the intermediary. As Gerardi, Hörner, & Maestri (2014) show, when the buyer's IR must be satisfied at every price, it is *with* loss to consider allocations with one price. So, there are outcomes attainable when the intermediary offers a menu of *lotteries over prices*, that are not attainable when each allocation in the menu can contain only one price. Formally, Gerardi, Hörner, & Maestri (2014) show that in this setting, it is without loss to focus on the following direct mechanisms: the intermediary maps each report θ to f_θ, a probability distribution over $\{0, 1\} \times \mathbb{R}_+$. Here, $f_\theta(p)$ is the probability of trade happening at price p, if θ is reported, and $f_\theta(0)$ is used to denote the probability of no trade. In the Appendix, I describe the class of mechanisms in detail.

[15] It can never be the case that the intermediary *strictly* reduces surplus, compared to when there is just a market. This is because the intermediary can always offer a menu with just allocation $(0, 0)$, so in equilibrium, it is as if there is no intermediary.

But why does offering lotteries over prices expand the set of attainable outcomes? The idea is that in equilibrium, prices contain information about the seller's type. When each allocation only contains a single price, the only information contained in this price is what types of the seller chose the allocation with this price in equilibrium. When the intermediary can map reports to lotteries over prices, it allows the intermediary greater flexibility in *how* to communicate the information elicited from the seller, to the buyer.

The above discussion might lead us to believe that it is this limited commitment on part of the intermediary that allows the market to disrupt its operation. However, with two types, if the prior satisfies the *lemons condition*, the presence of the market would *still* result in a breakdown:

Proposition 2. *Suppose there are two possible types, the lemons condition is satisfied, and the intermediary can offer a mechanism that maps reports to lotteries over prices. Then, the unique equilibrium outcome still involves only the lower type trading.*

The proof of the above proposition is in the Appendix. With more than two types, the analysis with lotteries over prices becomes quite complex, and therefore for tractability, I restrict attention to the case where any option in the menu has a single price. I should point out however, that with more that two types, it is still possible to argue that if types are *sufficiently* far apart, the presence of the market leads to a breakdown. However, getting a closed-form condition analogous to the BLC is difficult.

5 Conclusion

In this paper, I highlight the extent to which the presence of outside trading opportunities can disrupt intermediated trade. A seller who decides to trade through an intermediary, usually *also* has the option to sell her good without the intermediary. Selling without the intermediary can take several forms. For example, the seller can negotiate with a potential buyer directly. Or the intermediary could represent the legitimate channel of sale; if seller is unable to sell through this channel, she can sell on a "black market".

I model this outside selling opportunity as a static competitive market, where trade takes place at a single price. My main result is that under some conditions, the presence of the market completely destroys any efficiency gains from intermediated trade: in the unique equilibrium outcome, *only* the lowest type trades. The market "infects" the intermediary; in equilibrium, it is as if there is *just* a static, competitive market plagued with severe adverse selection, and no intermediary. I also provide conditions under which the the market results in inefficiency, but does not cause a total breakdown of trade.

In this paper, I go with a particular interpretation of how the intermediary operates. I model this as a bilateral trade setting, where the intermediary is brokering trade between a seller and a particular buyer, and this buyer *will* buy as long as he anticipates no loss from buying. The intermediary in my model can *commit* to randomising between offering and not offering the seller the seller the opportunity to trade. While this interpretation might make sense for some settings, in others, it might be unrealistic that the intermediary can commit to randomise.

However, even if the the intermediary cannot commit to randomise, a common feature of many settings is that there is some uncertainty associated with trade at higher prices. This uncertainty might arise because the intermediary has to "search" for potential buyers, and at a high price, it might not be able to find a buyer. This could be the case when potential buyers have identical payoff functions with respect to the good, but are heterogeneous in terms of wealth. For example, it could be that all buyers *want* to but at a high price, if *only* high quality goods are selling at higher prices in equilibrium, but most buyers are cash constrained. Therefore, at a high price, with some probability, the intermediary may not be able to find an appropriate buyer.

But even with this alternative interpretation, the key idea behind separation of high and low types remains the same: at higher prices, the probability of trade is lower. This suggests that my analysis of the disruption caused by the market goes though with alternative interpretations as well. I choose not to model the "search" for the buyer, or any other features of the setting that may give rise to the probability of trade-price trade off. I assume that the intermediary can commit to any menu. I show that *even* with this commitment power that allows the intermediary to create arbitrary trade offs between price and probability of trade, breakdown cannot be avoided under certain conditions. I leave the more general analysis of intermediation with outside selling opportunities for future work.

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6 Appendix

6.1 Notation

I first establish some notation. For any menu $\mathcal{M}$ offered by the intermediary, and any equilibrium of the game induced by this menu, let $\Theta_{(\pi,p),\mathcal{M}} = \{(\theta|\ \sigma((\pi,p)|\theta) > 0\}$, so $\Theta_{(\pi,p),\mathcal{M}}$ is the set of all types that in equilibrium, choose allocation $(\pi,p) \in \mathcal{M}$ with positive probability. Recall that $P_{\mathcal{M}} = \{p|\ \exists\ (\pi',p') \in \mathcal{M}\ with\ p' = p\}$: this is the set of all possible prices in the menu offered by the intermediary.

It is without loss to consider menus such that for any price p, there is at most one allocation in the menu with this price. Let $P_\theta = \{p|\ \sigma((\pi,p)|\theta) > 0\}$; this is the set of all prices such that on path, type θ chooses an allocation with this price with positive probability. For any on path price $p \in \bigcup_\theta P_\theta$, let $\mathbb{E}[\theta|p]$ denote the expected value of types that choose to sell at this price in equilibrium. Since this is an on path price, this Expectation is derived from the seller's strategy using Bayes Rule. Recall that the seller's strategy is given by $\sigma_S = (\sigma(.), \gamma_I(.), \gamma_M(.), \gamma'_M(.))$, where $\gamma_I : \Theta \times \mathcal{M} \to [0,1]$, and $\gamma_I((\pi,p),\theta)$ is the probability with which a seller of type θ chooses to sell through the intermediary, *conditional* on choosing (π,p) in the menu, *and* having the option to sell through the intermediary. So,

$$\mathbb{E}[\theta|p] = \frac{\sum_{\{\theta|p\in P_\theta\}} \mu(\theta)\sigma((\pi,p)|\theta)\gamma_I((\pi,p),\theta)(1+\alpha)\theta}{\sum_{\{\theta|p\in P_\theta\}} \mu(\theta)\sigma((\pi,p)|\theta)\gamma_I((\pi,p),\theta)}$$

For any $p \in P_{\mathcal{M}}$, $\pi_p(\theta)$ denotes the probability with which type θ sells at price p in equilibrium, and $\pi_M(\theta))$ denotes the probability with which θ sells on the market. So, for any p, if $(\pi,p) \in \mathcal{M}$ is the allocation in the menu with this price, then $\pi_p(\theta) = \pi\sigma((\pi,p)|\theta)\gamma_I((\pi,p),\theta)\sigma_B(p)$. Similarly, $\pi_M(\theta) = \sum_{\{(\pi,p)\in\mathcal{M}\}} \sigma((\pi,p)|\theta)\{\pi\gamma_M((\pi,p),\theta) + (1-\pi)\gamma'_M((\pi,p),\theta)\}$. Let Θ_+ be the set of all types that trade with positive probability in equilibrium. So, since $\sum_{P_{\mathcal{M}}} \pi_p(\theta) + \pi_M(\theta)$ is the total probability with which type θ trades in equilibrium, $\Theta_+ = \{\theta|\sum_{P_{\mathcal{M}}} \pi_p(\theta) + \pi_M(\theta) > 0\}$. Also, let $P_{(\mathcal{M},+)}$ be the set of all prices in the menu such that in equilibrium, trade happens at these prices with positive probability, through the intermediary. So, $P_{(\mathcal{M},+)} = \{p \in \mathcal{M}|\sum_\theta \pi_p(\theta) > 0\}$.

6.2 Two Useful Results

Before proving the Theorems, I state two results that will be useful for proving the Theorems. I provide the proof for these results at the end, after the proofs of the

Theorems.

We begin with a useful simplification. We can restrict attention to equilibria where the seller's strategy is such that for any θ, and any (π, p) such that $\sigma((\pi, p)|\theta) > 0$, $\gamma_I((\pi, p), \theta)$=1. So, it is without loss to restrict attention to equilibria in which if the seller chooses an allocation (π, p) with positive probability in equilibrium, then given the opportunity to sell through the intermediary at price p, the seller will do so with probability one. To state the result formally, let us define when two equilibria are outcome equivalent.

Fix any two menus $\mathcal{M}$, and $\mathcal{M}'$, and an equilibrium of the game induced by each of these menus. Let $P_{\mathcal{M},\mathcal{M}'} = P_{\mathcal{M}} \bigcap P_{\mathcal{M}'}$, the prices that are part of *both* menus. For the menu induced by $\mathcal{M}$, let the market price be given by p_M, and $\pi_p(\theta)$ denotes the probability with which type θ sells at price p in equilibrium, and $\pi_M(\theta))$ denotes the probability with which θ sells on the market. Similarly, for the menu induced by $\mathcal{M}'$, let the market price be given by p'_M, and $\pi'_p(\theta)$ denotes the probability with which type θ sells at price p in equilibrium, and $\pi'_M(\theta))$ denotes the probability with which θ sells on the market

Definition 3. *The two equilibria are outcome equivalent if i) $p_M = p'_M$, ii)in each equilibria, trade happens with positive probability at the same set of prices, that are in both menus, i.e. $P_{(\mathcal{M},+)} = P_{(\mathcal{M}',+)} \subseteq P_{\mathcal{M},\mathcal{M}'}$, and iii) for any θ, and any $p \in P_{\mathcal{M},\mathcal{M}'}$, $\pi_p(\theta) = \pi'_p(\theta)$, and $\pi_M(\theta)) = \pi'_M(\theta))$.*

Proposition 3. *Fix a menu $\mathcal{M}$, and an equilibrium of the game induced by this menu. Suppose there exists a θ, and a $(\pi, p) \in \mathcal{M}$ such that $\sigma((\pi, p)|\theta) > 0$, $\gamma_I((\pi, p), \theta) < 1$. Then we can construct another equilibrium, that is outcome equivalent to this equilibrium, where $\gamma'_I((\pi, p), \theta) = 1$*

Proof. If there exists a θ, and a $(\pi, p) \in \mathcal{M}$ such that $\sigma((\pi, p)|\theta) > 0$, $\gamma_I((\pi, p), \theta) < 1$, then it must be that $p = \theta$, otherwise it cannot be sequentially rational for θ to choose $\gamma_I((\pi, p), \theta) < 1$. This is because by Lemma 2, $p \geqslant p_M$, so $\gamma_I((\pi, p), \theta)$ cannot be lower than one because $p < p_M$. So, $\theta = p \geqslant p_M$. Consider the following modification to strategy of the seller of type θ: $\sigma'((\pi, p)|\theta) = \sigma((\pi, p)|\theta)\gamma_I((\pi, p), \theta)$, and $\gamma'_I((\pi, p), \theta) = 1$.

So, I modify type θ's strategy such that she chooses the allocation (π, p) with a strictly lower probability, and *conditional* on choosing (π, p) and having the opportunity to sell through the intermediary, it does so with probability one. The second modification in the strategy of type θ is that $\sigma'((0, 0)|\theta) = \sigma((0, 0)|\theta) + \sigma((\pi, p)|\theta)(1 - \gamma_I((\pi, p), \theta))$. Since I reduced the probability of θ choosing (π, p), the probability of θ choosing *some* other allocation in the menu must increase. I add this residual probability, $\sigma((\pi, p)|\theta)(1-\gamma_I((\pi, p), \theta))$, to $\sigma((0,0)|\theta)$, the allocation that represents *not* participating in the intermediary's trading process.

Once these modifications are made, we can find the appropriate $\gamma_M((\pi,p),\theta)$ and $\gamma_M^{'}((\pi,p),\theta)$ such that the *equilibrium outcome* remains same. This is because if $\theta = p$, then when it doesn't sell at p, it either sells on the market, or it does not sell at all. By increasing $\sigma((0,0)|\theta)$ to $\sigma^{'}((0,0)|\theta)$, and then making appropriate modifications to $\gamma_M((\pi,p),\theta)$ and $\gamma_M^{'}((\pi,p),\theta)$, we can make sure that the probability with which θ sells on the market is the same as the original equilibrium. Observe that under the modified strategies, the probability with which θ trades at p, through the intermediary, is also the same as in the original equilibrium. So, the equilibrium outcome remains the same as before. $\square$

I now state another result that will be useful for proving Theorem 1.

Proposition 4. *Fix a menu $\mathcal{M}$ and an equilibrium of the game induced by this menu, such that in this equilibrium, trade happens both through the intermediary and on the market with positive probability. Then, it must be that $(1+\alpha)\mathbb{E}[\theta^{'}|\theta^{'} \leqslant p_m] > p_M$.*

6.3 Proof of Theorem 1

Proof. I first consider equilibria in trade trade takes place with positive probability, *both* through the intermediary, and on the market. Therefore, if the intermediary, offers the menu $\mathcal{M}$, we have that $P_{(\mathcal{M},+)} \neq \varnothing$, *and* $\sum_\theta \pi_M(\theta) > 0$. I first show that p_M is uniquely determined in any such equilibrium.

Lemma 5. *When BLC is satisfied, then in any equilibrium where trade takes place with positive probability, both through the intermediary, and on the market, $p_M = (1+\alpha)\theta_n$.*

Proof. In any equilibrium, we must have $p_M \geqslant (1+\alpha)\theta_n$ in any equilibrium, as θ_n is the lowest type. Suppose $p_M > (1+\alpha)\theta_n$. Then it must be that in equilibrium, *some* $\theta > \theta_n$ trades on the market with positive probability. So, there exists a $k \geqslant (n-1)$ such that $k = \min\{k^{'} \geqslant (n-1)|p_M \geqslant \theta_{k'}\}$. So, θ_k denotes the highest type (recall that types with lower indices are higher), such that $p_M \geqslant \theta_k$. Therefore, $\mathbb{E}[\theta^{'}|\theta^{'} \leqslant p_M] = \mathbb{E}[\theta^{'}|\theta^{'} \leqslant \theta_k]$. From BLC, we know that $\mathbb{E}[\theta^{'}|\theta^{'} \leqslant \theta_k] < \theta_k$, since $k \geqslant (n-1)$. So, $\mathbb{E}[\theta^{'}|\theta^{'} \leqslant \theta_k] < p_M$ as well, since $p_M \geqslant \theta_k$. This is a contradiction to Proposition 4, since now, we have that $\mathbb{E}[\theta^{'}|\theta^{'} \leqslant p_M] < p_M$. Therefore, in equilibrium, we cannot have $p_M > (1+\alpha)\theta_n$, and p_M must be $(1+\alpha)\theta_n$ in any equilibrium.

$\square$

Proposition 5. *When BLC is satisfied, there can be no equilibrium where trade takes place with positive probability, both through the intermediary, and on the market, and a type $\theta > \theta_n$ trades with positive probability.*

Proof. Suppose there is such an equilibrium. By Lemma 5, we have $p_M = (1+\alpha)\theta_n$. I first argue that $p_M < \theta$ for *any* $\theta > \theta_n$. To see this, consider the lowest two types, θ_{n-1}, and θ_n. By BLC, we have that $(1+\alpha)\mathbb{E}[\theta'|\theta' \in \{\theta_{n-1}, \theta_n\}] < \theta_{n-1}$, which implies that $(1+\alpha)\theta_n < \theta_{n-1}$. Since $p_M < \theta_{n-1}$, it is also strictly lower than any other $\theta > \theta_n$. I divide the proof in steps:

Step 1: *Any type $\theta > \theta_n$ that trades with positive probability in equilibrium, must trade through the intermediary, at a price strictly greater than $p_M = (1+\alpha)\theta_n$.*

This follows directly from the fact that p_M is strictly lower than any $\theta > \theta_n$.

Step 2: *For any $p' \in P_{(\mathcal{M},+)}$, $p' > p_M$.*

By **Step 1**, $\theta > \theta_n$ can *only* trade through the intermediary, at a price strictly greater than p_M, so there exists an allocation $(\pi, p) \in \mathcal{M}$ such that $p > p_M$, and $p \in P_{(\mathcal{M},+)}$, i.e., there is some price that's strictly greater than p_M, at which trade takes place with positive probability. The claim in **Step 2** then follows from Lemma 2.

Step 3: *In equilibrium, there exists an allocation $(\pi', p') \in \mathcal{M}$, such that for θ_n, $\sigma((\pi', p')|\theta_n) =$ 1.*

Suppose θ_n chooses more than one allocation in $\mathcal{M}$ with positive probability in equilibrium. Then, θ_n must be indifferent between all these allocations. By Lemma 3, that the *effective type* of θ_n is $p_M = (1+\alpha)\theta_n$. So, if θ_n is indifferent between allocations (π', p'), and (π'', p''), such that $p' < p''$, and $\pi' > \pi''$, then it is *as if* the intermediary is operating in isolation, and there is a hypothetical type p_M, which is indifferent between these allocations:

$$\pi'(p' - p_M) = \pi''(p'' - p_M)$$

This implies that that for any $\theta > p_M$, (π'', p'') is strictly preferred to (π', p'), by Lemma 1. Since any $\theta > \theta_n$ is also strictly greater than p_M, therefore, in equilibrium, *no* $\theta > \theta_n$ will choose (π', p') with positive probability. But if only θ_n is selling at p', then the buyer wouldn't buy, as $p' > (1+\alpha)\theta_n$. This is a contradiction.

Step 4: *Let $(\pi', p') \in \mathcal{M}$ be such that for θ_n, $\sigma((\pi', p')|\theta_n) = 1$. Then, $\{\theta > \theta_n | \sigma((\pi', p')|\theta) > 0\} \neq \varnothing$. Also, if $\theta^{'} > \theta > \theta_n$, and $\sigma((\pi', p')|\theta^{'}), \sigma((\pi', p')|\theta) > 0$, then it must be the case that $\sigma((\pi', p')|\theta) = 1$.*

Since $p' > (1+\alpha)\theta_n$, therefore, for the buyer to buy at this price, some $\theta > \theta_n$ must choose the allocation (π', p') with positive probability in equilibrium. Suppose $\theta^{'} > \theta > \theta_n$ and $\sigma((\pi', p')|\theta^{'}) > 0$, and $\sigma((\pi', p')|\theta) < 1$. Then, there must be another allocation, $(\pi^{''}, p^{''})$, such that $\sigma((\pi^{''}, p^{''})|\theta) > 0$. Therefore, θ is indifferent between (π', p') and $(\pi^{''}, p^{''})$. Then, there can be two cases.

Either, $\pi' > \pi^{''}$, and $p' < p^{''}$. In this case, θ is indifferent, so $\theta^{'}$ would strictly prefer $(\pi^{''}, p^{''})$ to (π', p'), which is a contradiction, as $\sigma((\pi', p')|\theta^{'}) > 0$. The second case is that $\pi' < \pi^{''}$, and $p' > p^{''}$. In this case, θ_n strictly prefers $(\pi^{''}, p^{''})$, which is again a contradiction. So, we must have $\sigma((\pi', p')|\theta) = 1$.

From **Step 4**, we derive the desired contradiction to the fact that there exists an equilibrium, where trade takes place both through the intermediary and on the market, *and* types higher than θ_n trade with positive probability. By **Step 3**, in this equilibrium, there is an allocation (π', p') such that θ_n chooses this allocation with probability one. **Step 4** implies that there exists a highest type $\theta^* > \theta_n$, such that in equilibrium, $\sigma((\pi', p')|\theta^*) > 0$, and for all $\theta < \theta^*$, $\sigma((\pi', p')|\theta) = 1$. So, in equilibrium, conditional on price p', the maximum value of buyer's expected value for the good is $\mathbb{E}[\theta|\theta \leqslant \theta^*]$.[16] But, by BLC, $\mathbb{E}[\theta|\theta \leqslant \theta^*] < \theta^*$, which violates the IR for type θ^*. This is the desired contradiction, and completes the proof of Proposition 5.

□

Now suppose we look at equilibria where trade takes place only through the intermediary.

Proposition 6. *When BLC is satisfied, there is no equilibrium in which trade takes place only through the intermediary, and types $\theta > \theta_n$ trades with positive probability.*

Proof. Suppose by contradiction, that there exists an equilibrium where trade takes place only through the intermediary, and types $\theta > \theta_n$ trades with positive probability. Since no trade takes place on the market, p_M is determined by off path beliefs and does not have to be equal to $(1+\alpha)\theta_n$. But it must be that $p_M \geqslant (1+\alpha)\theta_n$. So, $\theta_n < p_M$.

[16] It need not be equal to this, as θ^* can randomise.

In this equilibrium, it must be that θ_n must choose an allocation (π', p') with probability one, such that $\pi' = 1$, and $p' > (1+\alpha)\theta_n$. π' must be one, because if $\pi' < 1$, then since $p_M > \theta_n$, θ_n *will* sell on the market if trade does not happen through the intermediary. But in equilibrium, no trade takes place on the market. Secondly, it must be that $p' > (1+\alpha)\theta_n$. This is because $\theta > \theta_n$ trade with positive probability in equilibrium, so there exists an allocation (π, p) such that $p \geqslant \theta_{n-1} > (1+\alpha)\theta_n$, and $\sigma_B(p) = 1$. Therefore, θ_n would never choose (π', p') if $p' = (1+\alpha)\theta_n$.

So, suppose θ_n chooses (π', p') with probability one in equilibrium. Since $p' > (1 = \alpha)\theta_n$, then, we can show, by using an argument like *Step 4* of Proposition 5, that there exits type θ^*, such that in equilibrium, θ^* is the highest type that chooses (π', p') with positive probability, and all $\theta < \theta^*$ choose (π', p') with probability one. Thus we have the desired contradiction: conditional on p', buyer's expected value for the good is at most $\mathbb{E}[\theta|\theta \leqslant \theta^*]$. But, by BLC, $\mathbb{E}[\theta|\theta \leqslant \theta^*] < \theta^*$, which violates the IR for type θ^*. This is the desired contradiction.

□

Propositions 5 and 6 complete the proof of sufficiency in Theorem 1. There can be two possible kinds of equilibria: where trade takes place both through intermediary and market, and where trade takes place only through the intermediary.[17] I show, by Propositions 5 and 6 respectively, that there can be no equilibrium of either kind where $\theta > \theta_n$ trade with positive probability, if the BLC is satisfied.

I now provide the proof of necessity: If the BLC is *not* satisfied, then there is always an equilibrium where $\theta > \theta_n$ trade with positive probability.

Proposition 7. *If the prior does not satisfy the BLC, there is always an equilibrium where types greater than θ_n trade with positive probability.*

Proof. I now construct such an equilibrium. Since the BLC is not satisfied, there exists a $\theta' > \theta_n$, such that $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta'] \geqslant \theta'$. Let $\theta^* = \max\{\theta' | (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta'] \geqslant \theta'\}$. Let $\theta^{**} = \min\{\theta|\theta > \theta^*\}$, so θ^{**} is the next higher type after θ^{**}. Observe that such a type always exists, since the prior satisfies the *lemons condition*, so θ^* cannot be θ_1. Now consider the following menu: $\mathcal{M} = \{(1, (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*])\}$.[18]

If the intermediary offers this menu, there exists an equilibrium where 1) No trade takes place on the market, 2) All $\theta \leqslant \theta^*$ trade with probability one and 3) $p_M = (1+\alpha)\theta_n$.

[17]Technically, there can also be equilibria where trade takes place *only* on the market. But when BLC is satisfied, in such equilibria, $\theta > \theta_n$ can never trade.

[18]Technically, $(0, 0)$ is always in the menu, but I don't write it explicitly here.

To see this, suppose $p_M = (1+\alpha)\theta_n$, and the buyer's strategy is $\sigma_B((1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]) = 1$. Clearly, $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*] > p_M$, so any type would prefer to sell at $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$, as opposed to p_M. Now observe that by definition of θ^*, it must be that $\theta^{**} > (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$. This is because $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^{**}]$ is a convex combination of $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$ and $(1+\alpha)\theta^{**}$. So, if $\theta^{**} \leqslant (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$, then $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^{**}] > \theta^{**}$, and this contradicts the fact that $\theta^* = \max\{\theta' | (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta'] \geqslant \theta'\}$. So, the set of types that sell at $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$ is $\{\theta \leqslant \theta^*\}$. Therefore, given p_M, and the buyer's strategy, types $\theta \leqslant \theta^*$ would choose to sell at price $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$. Since they are able to sell at this price with probability one, no trade takes place through the market. For the buyer, his strategy of buying at $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^*]$ is obviously optimal, given the seller's strategy. Lastly, $p_M = (1+\alpha)$ is determined by the off-path belief that if the seller is selling on the market, her type must be θ_n.

□

This completes the proof of Theorem 1.

□

6.4 Proof of Proposition 4

Proof. Suppose BLC is satisfied, and fix an equilibrium trade occurs with positive probability both through the intermediary and the market. There are two possible cases: $\Theta_+ = \{\theta' | \theta' \leqslant p_M\}$, i.e., there is *no* θ' strictly greater than p_M that trades with positive probability in equilibrium, or there exists a $\theta \in \Theta_+$ such that $\theta > p_M$. I consider these two cases separately, and show that in each case, we must have $\mathbb{E}[\theta'|\theta' \leqslant p_M] \geqslant p_M$.

Lemma 6. *Suppose $\Theta_+ = \{\theta' | \theta' \leqslant p_M\}$. Then, it must be that $\mathbb{E}[\theta'|\theta' \leqslant p_M] \geqslant p_M$.*

Proof. Suppose, by contradiction, that in equilibrium, $\mathbb{E}[\theta'|\theta' \leqslant p_M] < p_M$. Starting with with assumption, we will reach a contradiction. The proof proceeds in the following steps:

Step 1: $P_{(\mathcal{M},+)}$ *cannot contain more than one price.*

Suppose not, i.e., $P_{(\mathcal{M},+)}$ contains more than one price. By Lemma 2, $p' \geqslant p_M$ for all $p' \in P_{(\mathcal{M},+)}$. Let p, p'' be two prices in $P_{(\mathcal{M},+)}$, and without loss, let $p'' > p' \geqslant p_M$. By Lemma 2, this implies that $p' > p_M$ as well. Therefore, $p' > p_M$ for all $p' \in P_{(\mathcal{M},+)}$.

Since at any $p' \in P_{(\mathcal{M},+)}$, trade is taking place with positive probability (by definition of $P_{(\mathcal{M},+)}$), it must be that that for any such p', $\sigma_B(p') = 1$, i.e the buyer's strategy must be to buy at all these prices. Therefore, at any $p' \in P_{(\mathcal{M},+)}$, the buyer's interim IR must be satisfied. So, $(1+\alpha)\mathbb{E}[\theta'|p'] \geqslant p'$ for any $p' \in P_{(\mathcal{M},+)}$. Since $p' > p_M$ for any $p' \in P_{(\mathcal{M},+)}$, we have that $(1+\alpha)\mathbb{E}[\theta'|p'] > p_M$ for all $p' \in P_{(\mathcal{M},+)}$.

Since $p' > p_M$ for all $p' \in P_{(\mathcal{M},+)}$, any $\theta \in \Theta_+$ will sell on the market, only if she chooses an allocation in the intermediary's menu and does not get the option to trade through the intermediary. Therefore no type would choose to trade directly on the market, i.e. for all $\theta \in \Theta_+$, $\sigma((0,0)|\theta) = 0$. So, all types in the set $\{\theta'|\theta' \leqslant p_M\}$ choose *some* $p' \in P_{(\mathcal{M},+)}$ in equilibrium, i.e. $\sum_{p' \in P_{(\mathcal{M},+)}} \sigma((\pi',p')|\theta) = 1$. So, by law of total expectation, we have that

$$(1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant p_M] = \sum_{p' \in P_{(\mathcal{M},+)}} \sum_{\theta' \leqslant p_M} \mu(\theta')\sigma((\pi',p')|\theta')(1+\alpha)\mathbb{E}[\theta'|p']$$

But as we argued earlier, by the buyer's interim IR, $(1+\alpha)\mathbb{E}[\theta'|p'] > p_M$ for all $p' \in P_{(\mathcal{M},+)}$, so this implies that $(1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant p_M] > p_M$, which is a contradiction.

Step 2: *If $P_{(\mathcal{M},+)}$ is a singleton, $\{p'\}$, then $p' = p_M$.*

If $p'' > p_M$, by the buyer's interim IR, it must be that $(1+\alpha)\mathbb{E}[\theta'|p'] \geqslant p'' > p_M$. But then, as before, all types in $\Theta_+ = \{\theta'|\theta' \leqslant p_M\}$ will choose the allocation (π',p') with probability one. But this implies that $(1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant p_M] > p_M$, which is a contradiction. Now, p' cannot be strictly lower than p_M, so it must be that $p' = p_M$.

I now show that **Step 2** leads to a contradiction. The only possibility is that $P_{(\mathcal{M},+)} = \{p_M\}$. So, every $\theta \leqslant p_M$ is indifferent between trading directly on the market, i.e. choosing $(0,0)$ in $\mathcal{M}$, or choosing (π',p'), and selling on the market if trade does not happen through the intermediary. For any $\theta \leqslant p_M$, $\sigma((\pi',p'|\theta) + \sigma((0,0)|\theta) = 1$.

Observe that since $p' = p_M$, therefore, by the buyer's interim IR, $(1+\alpha)\mathbb{E}[\theta|p'] \geqslant p_M$. So, since $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant p_M] < p_M$, therefore, by Law of Total Expectation, we have that $(1+\alpha)\mathbb{E}[\theta|(0,0)] < p_M$, where $\mathbb{E}[\theta|(0,0)]$ is the expected value of the seller's type, conditional on choosing $(0,0)$ in the menu.

The price p_M on the market, is determined by the market clearing condition. Observe that for any $\theta < p_M$, $\gamma_M'((\pi',p'),\theta) = \gamma_M'((0,0),\theta) = 1$. If there is a $\theta = p_M$, then this type may randomise between selling and not selling on the market, but we assume that it always sells (nothing will change if we don't assume this, its just to simplify notation).

$$p_M = \frac{(1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')|\theta)(1+\alpha)\theta + \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((0,0)|\theta)(1+\alpha)\theta}{(1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')|\theta) + \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((0,0)|\theta)}$$

$$\implies \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((0,0)|\theta)\{p_M - (1+\alpha)\theta)\} = (1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')|\theta)\{(1+\alpha)\theta - p_M\}$$

$$\implies \sum_{\theta \leqslant p_M} \mu(\theta)(1-\sigma((\pi',p')|\theta))\{p_M-(1+\alpha)\theta)\} = (1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')|\theta)\{(1+\alpha)\theta-p_M\}$$

In the above equation, the RHS is equal to $(1-\pi')(\sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')|\theta))\{\mathbb{E}[\theta|p'] - p_M\}$, which is non negative, as $\mathbb{E}[\theta|p'] \geqslant p_M$. Now, consider the LHS:

$$\sum_{\theta \leqslant p_M} \mu(\theta)(1-\sigma((\pi',p')|\theta))\{p_M - (1+\alpha)\theta)\}$$

$$= \sum_{\theta \leqslant p_M} \mu(\theta)\{p_M - (1+\alpha)\theta)\} - \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')\{p_M - (1+\alpha)\theta)\}$$

$$= \{p_M - (1+\alpha)\mathbb{E}[\theta|\theta \leqslant p_M]\}(\sum_{\theta \leqslant p_M} \mu(\theta)) + \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')\{(1+\alpha)\theta - p_M\}$$

$$> \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')\{(1+\alpha)\theta) - p_M\}$$

$$\geqslant (1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')\{(1+\alpha)\theta) - p_M\}$$

This is because $p_M > (1+\alpha)\mathbb{E}[\theta|\theta \leqslant p_M]$, and $\sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi',p')\{(1+\alpha)\theta - p_M\} \geqslant 0$. Observe that the last expression is the RHS, so LHS is strictly greater than RHS, but this is a contradiction, as we started with LHS=RHS. This concludes the proof of Lemma 6. □

I now consider the case where there exists a $\theta > p_M$ in Θ_+.

Lemma 7. *Suppose there exists a $\theta \in \Theta_+$ such that $\theta > p_M$. Then, it must be that $\mathbb{E}[\theta'|\theta' \leqslant p_M] \geqslant p_M$.*

Proof. Suppose, by contradiction, that in equilibrium, $(1+\alpha)\mathbb{E}[\theta'|\theta' \leqslant p_M] < p_M$. Starting with with assumption, we will reach a contradiction. The proof proceeds in the following steps:

Step 1: *All allocations* (π'', p'') *that are chosen with positive probability in equilibrium must have* $p'' > p_M$.

Since $\theta > p_M$ trade with positive probability in equilibrium, there exists an allocation (π, p) such that $p > p_M$, and at p, $\sigma_B(p) = 1$. Therefore, by choosing (π, p), the seller can sell at $p > p_M$ with positive probability. Since such an allocation exists, therefore in equilibrium, *all* types must choose (π'', p'') with $p'' > p_M$.

Step 2: *In equilibrium, there must exists at least one allocation* (π', p') *which is chosen with positive probability by both* $\theta \leqslant p_M$*, and by* $\theta > p_M$.

Now, suppose there is no allocation that's chosen with positive probability by both $\theta \leqslant p_M$, and by $\theta > p_M$. So, any allocation that's chosen with positive probability in equilibrium, is either *only* chosen by $\theta \leqslant p_M$, or *only* chosen by $\theta > p_M$. Let $supp(\sigma)_{\theta \leqslant p_M} = \{(\pi, p) \in \mathcal{M} | \sigma((\pi, p)|\theta) > 0 \ for \ some \ \theta \leqslant p_M\}$. This is the set of all allocations chosen with positive probability by $\theta \leqslant p_M$. By **Step 1**, for any $(\pi, p) \in supp(\sigma)_{\theta \leqslant p_M}$, $p > p_M$. Therefore, we must have $(1 + \alpha)\mathbb{E}[\theta|p] > p_M$, to satisfy the buyer's interim IR. So, $\sum_{(\pi,p) \in supp(\sigma)_{\theta \leqslant p_M}} \sigma((\pi, p)|\theta) = 1$ for every $\theta \leqslant p_M$, and for every $(\pi, p) \in supp(\sigma)_{\theta \leqslant p_M}$, we have $(1 + \alpha)\mathbb{E}[\theta|p] > p_M$. Therefore, by Law of Total Expectation, we have $(1 + \alpha)\mathbb{E}[\theta|\theta \leqslant p_M] > p_M$. But this is a contradiction, since since $(1 + \alpha)\mathbb{E}[\theta|\theta \leqslant p_M]$.

Step 3: *There exists exactly one allocation* (π', p') *which is chosen with positive probability by both* $\theta \leqslant p_M$*, and by* $\theta > p_M$.

Suppose there is more than one such allocation. Recall that the *effective type* of all $\theta \leqslant p_M$ is p_M, so "type" p_M must be indifferent between all such allocations. But then, for any two allocations, if p_M is indifferent, then by Lemma 1 any $\theta > p_M$ must strictly prefer the allocation with the lower probability of trade and higher price. This contradicts the fact that *both* allocations are chosen with positive probability by both $\theta \leqslant p_M$, and by $\theta > p_M$.

Step 4: *Let (π', p') denote the allocation that's chosen with positive probability, both by types $\theta < p_M$, and by types $\theta > p_M$. Then $(1+\alpha)\mathbb{E}[\theta \leqslant p_M|p'] < p_M$, where $\mathbb{E}[\theta \leqslant p_M|p']$ denotes the expected value of the seller's type, conditional on being weakly lower than p_M, and choosing (π', p').*

This follows from Law of Total Expectation. Let $supp(\sigma)_{(\theta \leqslant p_M)}$ denote the set of allocation that's chosen by *only* types $\theta \leqslant p_M$ in equilibrium. So, the sell of *all* allocations chosen with positive probability by $\theta \leqslant p_M$ is given by $supp(\sigma)_{(\theta \leqslant p_M)} \bigcup (\pi', p')$.

Now, by **Step 1**, for any $(\pi, p) \in supp(\sigma)_{(\theta \leqslant p_M)}$, $p > p_M$. So, to satisfy the buyer's interim IR at p, we must have $(1+\alpha)\mathbb{E}[\theta|p] > p_M$. Recall that for any $\theta \leqslant p_M$ $\sum_{(\pi,p) \in supp(\sigma)_{(\theta \leqslant p_M)}} \sigma((\pi, p)|\theta) + \sigma((\pi', p')|\theta) = 1$. Now, the claim in **Step 4** follows from the Law of Total Expectation. Since $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant p_M] < p_M$, and $(1+\alpha)\mathbb{E}[\theta|p] > p_M$ for all $(\pi, p) \in supp(\sigma)_{(\theta \leqslant p_M)}$, we have that $(1+\alpha)\mathbb{E}[\theta \leqslant p_M|p'] < p_M$.

Now I show that **Step 4** results in a contradiction. To see this, recall that the market price p_M is determined by the market clearing condition in equilibrium. Observe that for any $\theta < p_M$, $\gamma_M'((\pi', p'), \theta) = \gamma_M'((\pi, p), \theta) = 1$, for any $(\pi, p) \in supp(\sigma)_{(\theta \leqslant p_M)}$. If there is a $\theta = p_M$, then this type may randomise between selling and not selling on the market, but we assume that it always sells (nothing will change if we don't assume this, its just to simplify notation). I now denote $supp(\sigma)_{(\theta \leqslant p_M)}$ by the shorthand notation $S_{(\leqslant)}$. So,

$$p_M = \frac{\sum_{S_{(\leqslant)}} (1-\pi) \sum_{\theta \leqslant p_M} \mu(\theta)\sigma(\pi, p)|\theta)(1+\alpha)\theta + (1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi', p')|\theta)(1+\alpha)\theta}{\sum_{S_{(\leqslant)}} (1-\pi) \sum_{\theta \leqslant p_M} \mu(\theta)\sigma(\pi, p)|\theta) + (1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi', p')|\theta)}$$

$$\implies (1-\pi') \sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi', p')|\theta)(p_M - (1+\alpha)\theta) = \sum_{S_{(\leqslant)}} (1-\pi) \sum_{\theta \leqslant p_M} \mu(\theta)\sigma(\pi, p)|\theta)((1+\alpha)\theta - p)$$

The RHS is equal to $\sum_{S_{(\leqslant)}} (1-\pi)(\sum_{\theta \leqslant p_M} \mu(\theta)\sigma((\pi, p)|\theta))\{\mathbb{E}[\theta|p] - p_M\}$, which strictly positive, as $\mathbb{E}[\theta|p] > p_M$ for every $(\pi, p) \in S_{(\leqslant)}$. From here, we can reach a contradiction in exactly the same manner as we do at the end of Lemma 6. To see this, first observe that $\pi' < \pi$, and $p' > p$ for all $(\pi, p) \in S_{(\leqslant)}$. This follows directly from Lemma 1, since in equilibrium, (π', p') is chosen by $\theta > p_M$ with positive probability, and any $(\pi, p) \in S_{(\leqslant)}$ is chosen by *only* $\theta \leqslant p_M$. This, in combination with the fact that $p_M > (1+\alpha)\mathbb{E}[\theta \leqslant p_M|p']$, will lead to LHS > RHS, which is the desired contradiction. This completes the proof.

□

6.5 Proof of Theorem 2

The proof broadly proceeds in the following steps. I first establish some properties of equilibria when the intermediary is operating alongside the market. Then, I will show that for any such equilibrium, there exists an ϵ, such that of $\tilde{\theta} - \theta < \epsilon$, then we can construct an equilibrium for when the intermediary is operating alone, that generates strictly higher surplus that this equilibrium. Finally, I will argue that since we can do this for any equilibrium, this must also be true for the surplus maximising equilibrium when the intermediary is operating alongside the market.

I first show that when the intermediary is operating alongside the market, all equilibria must have a particular form. Let $\theta^* = \min\{\theta | \theta > \tilde{\theta}\}$.

Lemma 8. *When the intermediary is operating alongside the market, in any equilibrium, $p_M \in [(1+\alpha)\theta_n, \theta^*)$. Therefore, in any equilibrium, only $\theta \leqslant \tilde{\theta}$ can trade on the market.*

Proof. This follows from Proposition 4. Recall that $\tilde{\theta}$ is the highest type θ' such that $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta'] \geqslant \theta'$. Since θ^* is the type immediately higher that $\tilde{\theta}$, by Proposition 4, it cannot be that $p_M \geqslant \theta^*$. Because if this is the case, then let $\theta^{**} \geqslant \theta^*$ be the maximum type that's (weakly) lower than p_M. Then $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant p_M] = (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^{**}]$, but $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^{**}] < \theta^{**}$ by definition of $\tilde{\theta}$. Also, $\theta^{**} \leqslant p_M$, so we have that $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \theta^{**}] < p_M$, which contradicts Proposition 4.

Therefore $p_M < \theta^*$. In any equilibrium, we must have $p_M \geqslant (1+\alpha)\theta_n$. So, $p_M \in [(1+\alpha)\theta_n, \theta^*)$. Since $\tilde{\theta} < (1+\alpha)\theta_n$, this implies that all types $\theta \leqslant \tilde{\theta}$ can trade on the market in equilibrium. And for any type $\theta' > \theta$, as I argued, it cannot be that $p_M \geqslant \theta'$. So, no such type can trade on the market in equilibrium. □

Therefore,in any equilibrium, all $\theta \leqslant \tilde{\theta}$ must trade with probability one. For any equilibrium, let $\Theta_{(+,>)}$ denote the set of types (if any) that are strictly greater than $\tilde{\theta}$, and trade with positive probability in equilibrium. If $\Theta_{(+,>)} \neq \varnothing$, let $\overline{\theta} = \max\{\theta | \theta \in \Theta_{(+,>)}\}$, i.e., $\overline{\theta}$ is the highest type that trades with positive probability in equilibrium. Also, recall that $\theta^* = \min\{\theta|\theta > \tilde{\theta}\}$, so θ^* is the lowest type in $\Theta_{(+,>)}$.

I now show that any equilibrium induces a *segmentation* of types in $\Theta_{(+,>)}$.

Lemma 9. *Suppose in equilibrium, $\Theta_{(+,>)} \neq \varnothing$. Then, there exists a partition of $\Theta_{(+,>)}$, denoted by $\{\theta^1, \theta^2, \ldots \theta^m\}$, where $\theta^* \leqslant \theta^1 < \theta^2 \ldots < \theta^m \leqslant \overline{\theta}$, where the ith segment is given* *by* $\Theta_i = \{\theta | \theta^{i-1} < \theta \leqslant \theta^i\}$[19] *In equilibrium, all types in the same segment choose the same allocation, and if* $i < i^{'}$*, then the* i*th segment trades with higher probability and at a lower price in equilibrium than the* $i^{'}$*th segment.*

Proof. This follows directly from Lemma 1. First, observe that since any $\theta \in \Theta_{(+,>)}$ is strictly greater than p_M, these types can only trade through the intermediary.

In equilibrium, if a type $\theta \in \Theta_{(+,>)}$ chooses an allocation (π, p), then it weakly prefers (π, p) to all other allocations that are chosen with positive probability in equilibrium. So for any allocation (π', p') such that $\pi' < \pi$, and $p' > p$, by Lemma 1, if θ weakly prefers (π, p) to (π', p'), then all $\theta^{'} < \theta$ strictly prefer (π, p) to (π', p'), and therefore no $\theta^{'}$ will choose (π', p') in equilibrium. Similarly, no $\theta^{'} > \theta$ will choose (π', p') with $\pi' > \pi$, and $p' < p$.

So, let (π^1, p^1) be the allocation chosen by θ^*, the lowest type in $\Theta_{(+,>)}$. Then no $\theta \in \Theta_{(+,>)}$ will choose an allocation with a lower price an higher allocation probability. Let $\theta^{''}$ be the highest type that chooses (π^1, p^1) in equilibrium. The fact that all $\theta^* < \theta < \theta^{''}$ also choose (π^1, p^1) in equilibrium follows from Lemma 1, since both $\theta^* < \theta$, and $\theta^{''} > \theta$ weakly prefer (π^1, p^1) to all other allocations. Therefore, $\theta^{''} = \theta^1$, and $\{\theta | \theta^* \leqslant \theta^{''}\}$ constitute the lowest segment in $\Theta_{(+,>)}$. Similarly, we can argue for the higher segments. □

Now, fix an equilibrium when the intermediary is operating alongside the market. I will consider two cases and show that in each case, I can construct an equilibrium for when the intermediary is operating is isolation, which has strictly higher expected surplus from trade.

Lemma 10. *Suppose* $\Theta_{(+,>)}$ *is empty, so only* $\theta \leqslant \tilde{\theta}$ *trade in equilibrium. Then, there exists an equilibrium with strictly higher surplus from trade than this one when there is no market.*

Proof. Suppose the intermediary is operating in isolation. Consider the menu $\mathcal{M} = \{(1, (1+\alpha)\theta), (\pi_H, \theta^*)$, where $\pi_H \in (0, 1)$. Without going into the argument in detail (such arguments appear elsewhere in the paper), I claim that if π_H is low enough, there exists an equilibrium where $\theta \leqslant \tilde{\theta}$ trade with probability one, and type θ^* trades with probability π_H. In this equilibrium, θ^* chooses allocation (π_H, θ^*) and all $\theta \leqslant \tilde{\theta}$ choose $(1, (1+\alpha)\theta)$. Obviously, such an equilibrium results in strictly higher surplus than the original equilibrium, with the market because now, θ^* is also trading with positive probability. □

[19]if $i = 1$, then $\theta^{i-1} = \theta^*$.

Lemma 11. *Now suppose $\Theta_{(+,>)} \neq \emptyset$. Here too, there exists an equilibrium with strictly higher surplus from trade than this one when there is no market.*

Proof. As I showed in Lemma 9, such an equilibrium consists of a partition of $\Theta_{(+,>)}$. Let the segmentation of $\Theta_{(+,>)}$ in this equilibrium be given by $\{\theta^1, \theta^2 \ldots, \theta^m\}$, and let allocation chosen by segment Θ_i be denoted by (π^i, p^i). First, I fix the segmentation $\{\theta^1, \theta^2 \ldots, \theta^m\}$, and provide an upper bound for the expected surplus from trade in equilibrium with this $\Theta_{(+,>)}$ and segmentation. After establishing this upper bound, I argue that this upper bound, and therefore the surplus in the original equilibrium, can be improved upon when there is no market.

In the original equilibrium, types $\theta \leqslant \tilde{\theta}$, all have an effective type that's $p_M \geqslant (1+\alpha)\theta_n$. By **Step 3** of Lemma 7, there is an allocation (π, p) that's chosen by only types $\theta \leqslant \tilde{\theta}$ in equilibrium. So, it must be that effective type p_M weakly prefers (π, p) to (π^1, p^1), the allocation chosen by Θ_1, the lowest segment of $\Theta_{(+,>)}$. This puts an upper bound on π^1, and therefore an upper bound on the probabilities of trade of all subsequent segments, since the highest type in any segment Θ_i weakly prefers (π^i, p^i) to (π^{i+1}, p^{i+1}).

The upper bound on π^1 is given by $\frac{(1+\alpha)\tilde{\theta}-(1+\alpha)\theta_n}{p^1-(1+\alpha)\theta_n}$. This is because π^1 is highest when effective type p_M is indifferent between (π, p) and (π^1, p^1). Since (π, p) is chosen by only types $\theta \leqslant \tilde{\theta}$, the maximum value of p is $(1+\alpha)\tilde{\theta}$. Also, the lowest value of p_M is $(1+\alpha)\theta_n$. So, the highest value that effective type p_M can get in equilibrium, is $(1+\alpha)\tilde{\theta} - (1+\alpha)\theta_n$. The better off type p_M is, the higher we can make π^1, if we keep p^1 fixed. After this, we can inductively modify the probability of trade of each segment accordingly, so that the highest θ in any segment Θ_i is indifferent between choosing (π^i, p^i), with the modified π^i, or between (π^{i+1}, p^{i+1}).

Now I argue that we can construct an equilibrium for the case where the intermediary operates in isolation, and the same segmentation of $\Theta_{(+,>)}$. To see this, observe that we can keep the segmentation fixed, and modify the probabilities of trade such that types $\theta \leqslant \tilde{\theta}$ trade with probability one at price $(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \tilde{\theta}]$. Unlike the case when the market is present, now, we need to make $\tilde{\theta}$ indifferent between $(1, (1+\alpha)\mathbb{E}[\theta|\theta \leqslant \tilde{\theta}])$, and (π^1, p^1), so if we keep p^1 the same as the original equilibrium, $\pi^1 = \frac{(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \tilde{\theta}]-\tilde{\theta}}{p^1-\tilde{\theta}}$. It is easy to see that if $\tilde{\theta}$ is close enough to θ_n, then, $\frac{(1+\alpha)\mathbb{E}[\theta|\theta \leqslant \tilde{\theta}]-\tilde{\theta}}{p^1-\tilde{\theta}} > \frac{(1+\alpha)\tilde{\theta}-(1+\alpha)\theta_n}{p^1-(1+\alpha)\theta_n}$, the upper bound derived for the equilibrium with the market. If π^1 is strictly greater, the probability of trade of all subsequent segments can be made strictly higher. ∎

This completes the proof. For any equilibrium in the presence of a market, we can construct an improvement. Therefore, whatever the surplus maximising equilibrium is, we can construct an improvement over that as well.

□

6.6 Proof of Proposition 2

I first describe the setup and the class of mechanisms I consider formally. As in the two type example, there are two types, θ_L and θ_H, where $\theta_L < \theta_H$, and the *lemons* condition holds: $(1+\alpha)\mathbb{E}[\theta] < \theta_H$. A mechanism is denoted by (M, f, P), where M is the set of messages, P is the set of "possible" prices at which trade can take place through the intermediary, and $f : M \to \Delta(P)$, so each report is mapped to a *lottery* over prices. I use $f_m(p)$ to denote the probability with which the seller is offered the chance to sell at price p if she reports m, and for any $m \in M$, $\sum_{p \in P} f_m(p) \leqslant 1$, where the weak inequality captures the fact that with some probability, the seller may not be offered the opportunity to sell through the intermediary. I assume that P is finite, but relaxing this assumption will not change the result in Proposition 2.

The timeline for the game is the same as before: 1) the intermediary offers a mechanism, 2) the seller makes a report to the mechanism, 3) the uncertainty associated with the mechanism is resolved; the seller learns whether or not she can trade through the intermediary, and if so, at what price, and 4) the seller decides where to sell, given her choices. In this setting, when the intermediary operates alongside the market, one can show, using standard arguments that the Revelation Principle holds, and it is without loss to restrict attention to *direct mechanisms*, so $M = \{\theta_H, \theta_L\}$. A mechanism is *IC* if, in the game induced by the mechanism, each type of the seller finds it optimal to report truthfully. As before, the buyer's interim IR must be satisfied, i.e., for trade to take place at any price in equilibrium, the buyer must find it optimal to buy at that price, given her beliefs.

I now show that there is *no* direct, *IC* mechanism, such that θ_H trades with positive probability in the game induced by that mechanism. To this end, I first argue that $p_M = (1+\alpha)\theta_L$ in any equilibrium.

Lemma 12. *For any mechanism, and any equilibrium of the game induced by this mechanism, $p_M = (1+\alpha)\theta_n$.*

Proof. Fix a mechanism, and an equilibrium induced by the mechanism. For any type θ, let $\pi_p(\theta)$ denote the probability with which type θ sells through the intermediary, at price p, and $\pi_M(\theta)$ denote the probability with which θ sells on the market, in equilibrium. Therefore:

$$p_M = \frac{\mu(\theta_H)\pi_M(\theta_H)(1+\alpha)\theta_H + \mu(\theta_L)\pi_M(\theta_L)(1+\alpha)\theta_L}{\mu(\theta_H)\pi_M(\theta_H) + \mu(\theta_L)\pi_M(\theta_L)}$$

Suppose $p_M > (1+\alpha)\theta_L$. Then, it must be that $\pi_M(\theta_H) > 0$. This in turn implies that $p_M \geqslant \theta_H$, because p_M must satisfy θ_H's IR. By the *lemons condition*, $p_M \geqslant \theta_H$ implies that $\pi_M(\theta_H) > \pi_M(\theta_L)$. I now make two observations. Firstly, for the seller of type θ_L, the option to sell on the market at $p_M \geqslant \theta_H$ exists, so she will never sell through the intermediary at at $p < \theta_H$. For trade to take place at p, the buyer's interim IR must be satisfied, so if $p \geqslant \theta_H$, then $\pi_p(\theta_H) > \pi_p(\theta_L)$.

The second observation is that because $p_M > \theta_L$, so since the seller can always sell on the market, θ_L will sell with probability one *overall*, across the mechanism and the market. So, if P is the set of all prices in the mechanism, we have:

$$\sum_{p \in P} \pi_p(\theta_L) + \pi_M(\theta_L) = 1 \tag{4}$$

Combined with the first observation, this leads to a contradiction. This is because $\pi_p(\theta_H) > \pi_p(\theta_L)$ for every p at which θ_L trades though the intermediary, and $\pi_M(\theta_H) > \pi_M(\theta_L)$, so 4 implies that $\sum_{p \in P} \pi_p(\theta_H) + \pi_M(\theta_H) > 1$. This cannot be the case because $\sum_{p \in P} \pi_p(\theta_H) + \pi_M(\theta_H)$ is the total probability of trade of type θ_H across the intermediary and the mechanism, and cannot exceed one.

□

So, I have shown that in any equilibrium, $p_M = (1+\alpha)\theta_L$. I will now argue that there exits no *IC* mechanism that induces an equilibrium where θ_H trades with positive probability.

Lemma 13. *There is no equilibrium where θ_H trades with positive probability.*

Proof. Fix a mechanism offered by the intermediary, and an equilibrium induced by this mechanism. Suppose, by contradiction, that θ_H trades with positive probability in equilibrium.

Firstly, observe that θ_H can only trade through the intermediary, because the *lemons condition* implies that $\theta_H > p_M$. For any $p \in P$, such that $f_{\theta_H}(p) > 0$, it must be that $p \geqslant \theta_H$, to satisfy the high type's IR. For sale to happen at any such p, it must also be the case that the buyer's interim IR at p is satisfied. So, if $p \geqslant \theta_H$, and the buyer's strategy is to buy at this price in equilibrium, then we must have:

$$\frac{\mu(\theta_L)f_{\theta_L}(p)(1+\alpha)\theta_L + \mu(\theta_H)f_{\theta_H}(p)(1+\alpha)\theta_H}{\mu(\theta_L)f_{\theta_L}(p) + \mu(\theta_H)f_{\theta_H}(p)} \geqslant p \geqslant \theta_H \tag{5}$$

5 says that the expected value of the good for the buyer, conditional on price p, must be at least θ_H. Let $P_H \subseteq P$ be the set of all prices p such that $p \geqslant \theta_H$, and in equilibrium, trades takes place with positive probability at p. So, for any $p \in P_H$, 5 is satisfied. Because of the lemons condition, it must therefore be that $f_{\theta_L}(p) < f_{\theta_H}(p)$ for any $p \in P_H$. So, if there is a price $p \geqslant \theta_H$, then report θ_H is mapped to that price with strictly higher probability that report θ_L. Also, by the buyer's interim IR, any price not in P_H must be weakly lower than $(1+\alpha)\theta_L$, since *only* θ_L is being mapped to that price. This contradicts the fact that the mechanism is *IC*. Because type θ_L, in expectation, would get a strictly higher payoff if she reports θ_H.

□

6.7 Market Can Improve Surplus

I now provide a three-type example of how the the presence of the market can sometimes *obfuscate* information contained in prices in a way that improves surplus.

Proposition 8. *Consider a setting with three types $\{\theta_1, \theta_2, \theta_3\}$, where $\theta_1 > \theta_2 > \theta_3$, and the probability of type θ is denoted by $\mu(\theta)$. Suppose the prior $\mu(.)$ satisfies the following:*

- *The lemons condition: $(1+\alpha)\mathbb{E}[\theta] < \theta_1$*
- $(1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}] > \theta_2$*, and* $\theta_2 \geqslant (1+\alpha)\theta_3$
- $(1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_1, \theta_2\}] < \theta_1$

Then, there exists an equilibrium in the presence of the market that results in strictly higher surplus than any equilibrium when the intermediary is operating alone.

Proof. Under the conditions on parameters, when the intermediary is operating in isolation, the surplus maximising equilibrium pools θ_2 and θ_3, and separates θ_1. I omit the proof for this here, but the idea is that the only other options are separating θ_3, and pooling θ_1 and θ_2, and separating all three types. The first out of these is not feasible, because of the the condition $(1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_1, \theta_2\}] < \theta_1$, and some straightforward algebra shows that the second is never optimal.

The surplus maximising menu consists of two allocations; it is given by $\mathcal{M}^{**} = \{(\pi_1, p_1), (\pi_{\{2,3\}}, p_{\{2,3\}})\}$, where $\pi_{\{2,3\}} = 1, p_{\{2,3\}} = \mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}]$, $p_1 = \theta_1$, and $\pi_H = \frac{p_{\{2,3\}} - \theta_2}{p_1 - \theta_2}$. If the intermediary offers this menu, then there exists an equilibrium in which:

- both θ_2 and θ_3 choose allocation $(\pi_{\{2,3\}}, p_{\{2,3\}})$, and trade with probability one, at price $p_{\{2,3\}}$.

- θ_1 chooses allocation (π_1, p_1), and trades with probability $\pi_1 \in (0,1)$, at price p_1.

In this equilibrium, π_1, the probability of trade of θ_1, is such that type θ_2 is indifferent between the two allocations. Also, observe that $p_{\{2,3\}}$, the price that θ_2 and θ_3 get, is is equal to $(1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}]$; this is the *maximum* possible price these two types can get if they are selling at one price, otherwise the buyer's interim *IR* will not be satisfied.

If somehow, we could increase the price in the allocation that θ_2 and θ_3 are choosing, to $p'_{\{2,3\}} > p_{\{2,3\}}$, then to make θ_2 indifferent between the two allocations, now, ${\pi_1}' = \frac{p'_{\{2,3\}} - \theta_2}{p_1 - \theta_2}$, which is strictly greater than π_1. Thus, we would be able to increase surplus, because θ_2 and θ_3 are still trading with probability one, and θ_1 is trading at a strictly higher probability. The idea is that if θ_2 and θ_3 are able to trade at a strictly higher price, their payoff from choosing the other allocation decreases, and we can increase the probability of trade for that allocation till θ_2 is indifferent again.

This is exactly what the market helps with. The intuition is the following: with the menu $\mathcal{M}^{**}$, only θ_1 chooses (π_1, p_1), and since $p_1 = \theta_1$, the expected value for the good for the buyer, conditional on p_1, is $(1+\alpha)\theta_1 > p_1$. So, there is some "room" here, in the sense that if θ_2 and θ_3 were *also* choosing this allocation with a (low enough) strictly positive probability, then the buyer's interim IR would still be satisfied at p_1.

he presence of the market helps exploit this room. Consider the following menu: $\mathcal{M}' = \{(1, p''), (\pi'', p''_1)\}$, where $p''_1 = \theta_1$, $\pi'' = \frac{p'' - \theta_2}{\theta_1 - \theta_2}$, and $p'' = \frac{(1+\alpha)[\mu(\theta_2)(1-\sigma_2)\theta_2 + \mu(\theta_3)(1-\sigma_3)\theta_3]}{\mu(\theta_2)(1-\sigma_2)+\mu(\theta_3)(1-\sigma_3)}$, where σ_2, σ_3 are such that $\frac{(1+\alpha)[\mu(\theta_2)\sigma_2\theta_2 + \mu(\theta_3)\sigma_3\theta_3]}{\mu(\theta_2)\sigma_2 + \mu(\theta_3)\sigma_3} = \theta_2$, and $(1+\alpha)\mathbb{E}[\theta|p''_1] \geqslant \theta_1$. Then, the following is an equilibrium:

- $p_M = \frac{(1+\alpha)[\mu(\theta_2)\sigma_2\theta_2 + \mu(\theta_3)\sigma_3\theta_3]}{\mu(\theta_2)\sigma_2 + \mu(\theta_3)\sigma_3} = \theta_2$

- Given p_B, θ_2, and θ_3 are indifferent between the two allocations, and randomise between them as part of their equilibrium strategy.

- Strategy for θ_2: $\sigma((\pi'', \theta_1)|\theta_2) = \sigma_2$, and $\sigma((1, p'')|\theta_2) = 1 - \sigma_2$.

- Strategy for θ_3: $\sigma((\pi'', \theta_1)|\theta_3) = \sigma_3$, and $\sigma((1, p'')|\theta_3) = 1 - \sigma_3$.

- θ_1 chooses (π'', θ_1) with probability one

- For θ_2 and θ_3, if they choose (π'', θ_1), and the opportunity to trade through intermediary *not* realise, they sell on the market.

Observe that $p'' > (1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}]$ by Law of Iterated Expectations, since since $(1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}] > \theta_2$, and $\frac{(1+\alpha)[\mu(\theta_2)\sigma_2\theta_2+\mu(\theta_3)\sigma_3\theta_3]}{\mu(\theta_2)\sigma_2+\mu(\theta_3)\sigma_3} = \theta_2$. So, $p'' > p_{\{2,3\}}$. It is easy to see that given the strategies of θ_2 and θ_3, the buyer's interim IR is satisfied at p''. Also, in equilibrium, θ_2 and θ_3 trade on the market only if they choose allocation (π'', θ_1), and can't trade through the intermediary. So, p_M is indeed $\frac{(1+\alpha)[\mu(\theta_2)\sigma_2\theta_2+\mu(\theta_3)\sigma_3\theta_3]}{\mu(\theta_2)\sigma_2+\mu(\theta_3)\sigma_3} = \theta_2$. Given p_B, the strategies are also sequentially rational. So, this is an equilibrium.

Since $p'' > p_{\{2,3\}}$, we have that $\pi'' > \pi_{\{2,3\}}$. So, in this equilibrium, both θ_2 and θ_3 still trade with probability one overall, and θ_1 trades with strictly higher probability than before. So, the expected surplus from trade is strictly higher than in the optimal equilibrium when the intermediary was operating in isolation.

So how exactly does the market help? With the market, in the equilibrium we constructed with menu $\mathcal{M}'$, the buyer's interim IR is satisfied at $p'' > (1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}] > \theta_2$ because of the manner in which θ_2 and θ_3 randomise between the two allocations. It is important to note that *both* θ_2 and θ_3 cannot be indifferent between the two allocations in $\mathcal{M}'$. This is because in absence of the market, for any two allocations, if θ_2 is indifferent between them, then θ_3 strictly prefers the one with the higher allocation probability. So, without the market, it cannot be that in equilibrium, *both* these types randomise between the *same* two allocations. So, with menu $\mathcal{M}^{**}$, θ_2 is indifferent and can randomise, but if only θ_2 randomises, the expected value conditional on $p_{\{2,3\}}$ becomes strictly lower than $(1+\alpha)\mathbb{E}[\theta|\theta \in \{\theta_2, \theta_3\}]$, which defeats the purpose of trying to increase $p_{\{2,3\}}$ while satisfying the buyer's interim IR.

Now, let us come back to the case where the market is present and the intermediary offers the menu $\mathcal{M}'$. Consider the equilibrium I constructed. Since $p_M = \theta_2$, the *effective* type of both θ_2 and θ_3 is θ_2, and it is indeed optimal for both of them to randomise between the two allocations in $\mathcal{M}'$. This completes the discussion for why the market can improve surplus.

$\square$